Sergey Izraylevich Vadim Tsudikman

SYSTEMATIC OPTIONS TRADING

Evaluating, Analyzing, and Profiting from Mispriced Option Opportunities

Systematic Options Trading

This page intentionally left blank

Systematic Options Trading

Evaluating, Analyzing, and Profiting from Mispriced Option Opportunities

Sergey Izraylevich and Vadim Tsudikman Vice President, Publisher: Tim Moore Associate Publisher and Director of Marketing: Amy Neidlinger Executive Editor: Jim Boyd Editorial Assistant: Pamela Boland Operations Manager: Gina Kanouse Senior Marketing Manager: Julie Phifer Publicity Manager: Laura Czaja Assistant Marketing Manager: Megan Colvin Cover Designer: Gary Adair Managing Editor: Kristy Hart Project Editors: Jovana San Nicolas-Shirley and Andy Beaster Copy Editor: Apostrophe Editing Services Proofreader: Williams Woods Publishing Services Indexer: Word Wise Publishing Srevices Senior Compositor: Gloria Schurick Manufacturing Buyer: Dan Uhrig

© 2011 by Pearson Education, Inc. Publishing as FT Press Upper Saddle River, New Jersey 07458

This book is sold with the understanding that neither the authors nor the publisher is engaged in rendering legal, accounting, or other professional services or advice by publishing this book. Each individual situation is unique. Thus, if legal or financial advice or other expert assistance is required in a specific situation, the services of a competent professional should be sought to ensure that the situation has been evaluated carefully and appropriately. The author and the publisher disclaim any liability, loss, or risk resulting directly or indirectly, from the use or application of any of the contents of this book.

FT Press offers excellent discounts on this book when ordered in quantity for bulk purchases or special sales. For more information, please contact U.S. Corporate and Government Sales, 1-800-382-3419, corpsales@pearsontechgroup.com. For sales outside the U.S., please contact International Sales at international@pearson.com.

Company and product names mentioned herein are the trademarks or registered trademarks of their respective owners.

All rights reserved. No part of this book may be reproduced, in any form or by any means, without permission in writing from the publisher.

Printed in the United States of America

First Printing September 2010

ISBN-10: 0-13-708549-4

ISBN-13: 978-0-13-708549-1

Pearson Education LTD. Pearson Education Australia PTY, Limited. Pearson Education Singapore, Pte. Ltd. Pearson Education North Asia, Ltd. Pearson Education Canada, Ltd. Pearson Education de Mexico, S.A. de C.V. Pearson Education—Japan Pearson Education Malaysia, Pte. Ltd. Library of Congress Cataloging-in-Publication Data

Izraylevich, Sergey, 1966Systematic options trading : evaluating, analyzing, and profiting from mispriced option opportunities / Sergey Izraylevich and Vadim Tsudikman.
p. cm.
ISBN-13: 978-0-13-708549-1 (hardback : alk. paper)
ISBN-10: 0-13-708549-4
Stock options. 2. Options (Finance) 3. Investment analysis. 4. Portfolio management. I. Tsudikman, Vadim, 1965- II. Title.
HG6042.197 2011
332.63'2283—dc22

This book is dedicated to Professor Uri Gerson, Hebrew University of Jerusalem.

This page intentionally left blank

Contents

.....

	Introduction
PART I	Criteria as the Basis of a Systematic Approach
chapter 1	General Presentation and Review of Criteria Properties \ldots 3
	1.1 The Main Tool for Solving the Selection Problem 3
	1.2 Formal Definition4
	1.3 Philosophy of Criteria Creation5
	1.4 Mission Fulfilled by Criteria6
	1.5 Forecast as a Key Element of the Criterion 8
	1.6 Classification of Criteria9
	1.6.1 Universal Criteria9
	1.6.2 Specific (Nonuniversal) Criteria
chapter 2	Review of the Main Criteria13
	2.1 Criteria Based on Lognormal Distribution13
	2.1.1 Description of Lognormal Distribution13
	2.1.2 Expected Profit on the Basis of Lognormal
	Distribution
	2.1.3 Profit Probability on the Basis of Lognormal
	Distribution
	2.2 Criteria Based on Empirical Distribution
	2.2.1 Description of Empirical Distribution 22
	2.2.2 Expected Profit on the Basis of Empirical
	Distribution
	2.2.3 Profit Probability on the Basis of Empirical
	Distribution

	2.2.4 Simplified Calculation Algorithm
	2.2.5 Modifications of Empirical Distribution31
	2.3 Criteria Based on the Ratio of Expected
	Profit to Loss
	2.3.1 Basic Concept and Criteria Calculation
	Method
	2.3.2 Criteria Calculation Example
	2.4 Criteria Based on Expert Distribution
	2.4.1 Basic Concept and Criteria Calculation
	Method
	2.4.2 Set of Standard Distributions
	2.4.3 Combining Separate Standard Distributions
	into a Unified Probability Density Function 45
	2.4.4 Criteria Calculation on the Basis of the Unified
	Probability Density Function
	2.4.5 Construction and Valuation of Complex
	Strategies Based on the Unified Probability
	Density Function 48 2.5 Source Channel 50
	2.5 Specific (Nonuniversal) Criteria
	2.5.1 Break-Even Range
	2.5.2 IV/HV Ratio
	2.5.3 Relative Frequency Criterion
	2.5.4 The Ratio of Normalized Time Value to the
	Coefficient of Absolute Price Changes Distribution58
ahantar O	
chapter 3	Evaluation of Criteria Effectiveness633.1 Introduction63
	3.2 Methods of Criteria Effectiveness Evaluation 64
	3.2.1 Correlation Between a Criterion and
	Profit as the Main Effectiveness Indicator
	3.2.2 Transformation of the Criteria Effectiveness
	Indicator
	3.2.3 The Dynamics of Transformed Effectiveness
	Indicators
	3.2.4. Selection of the Averaging Period

	3.3 Peculiarities of Criteria Effectiveness Evaluation 75
	3.3.1 Number of Combinations Used in
	<i>the Analysis</i>
	3.3.2 <i>Expressing Profit</i>
	3.3.3 Expressing Effectiveness Indicators
	3.4 Review of Criteria Effectiveness Indicators
	3.4.1 Correlation Between Criterion and
	Profit Values
	3.4.2 Correlation Between a Criterion and
	Profit Indexes86
	3.4.3 Correlation Between the Sharpe Ratios of
	Criterion and Profit
	3.4.4 Areas Ratio
	3.4.5 Other Effectiveness Indicators
	3.5 Summary 102
PART II	The Main Areas of Criteria Application
chapter 4	Selection of Option Combinations
	4.1 Introduction
	4.2 Analysis of Criteria Effectiveness in the Selection
	of Option Combinations106
	4.3 Factors That Affect Option Combinations
	Selection
	4.3.1 Absolute Values of the Criterion
	4.3.2 Strategy and Underlying Assets 112
	4.3.3 Simultaneous Analysis of Factors Affecting
	Combinations Selection 113
	4.4 Multistrategy, Long-Term Evaluation of Criteria
	Effectiveness
	4.4.1 Methods 115
	4.4.2 Results
	4.5 Summary 119
chapter 5	Selection of Option Strategies
	5.1 Introduction
	5.2 Evaluation of Criterion Effectiveness by Ranking
	Analysis

	5.2.1 Methods of Ranking Analysis
	5.2.2 Results of Ranking Analysis 131
	5.2.3 Generalized Ranking Analysis and
	Introduction of the Threshold Parameter 139
	5.2.4 Results of Generalized Ranking Analysis 140
	5.2.5 Maximum Obtainable Values of the
	Criterion Effectiveness Coefficient
	5.3 Traditional Methods of Evaluating the Criterion Effectiveness
	5.4 Synthetic Approach to Criterion Effectiveness
	Analysis
	5.5 The Model for Optimizing the Threshold
	Parameter
	5.6 Summary 159
chapter 6	Selection of Underlying Assets
	6.1 Introduction
	6.2 Analysis of Criteria Effectiveness in Selection
	of Underlying Assets 162
	6.3 Multistrategy, Long-Term Evaluation of
	Criteria Effectiveness
	6.3.1 Methods 166
	6.3.2 Results 167
	6.4 The Optimization Model for the Number of
	Underlying Assets 168
	6.4.1 Utility Indicators 169
	6.4.2 Utility Functions 171
	6.4.3 Convolution of Utility Functions and Deriving
	Optima for Different Strategies and Criteria 173
	6.5 Summary 176

PART III	Multicriteria Analysis
chapter 7	Basic Concepts of Multicriteria Selection as
	Applied to Options
	7.1 Introduction
	7.2 The Pareto Set
	7.2.1 The Algorithm of Forming the Pareto Set 183
	7.2.2 Widening the Pareto Set and the "Layer"
	Notion
	7.3 Convolution
	7.4 Comparative Analysis of Multicriteria and
	Monocriterion Selection Effectiveness
	7.5 Comparative Analysis of Two Multicriteria
	Selection Methods: Pareto Versus Convolution 197
	7.6 Summary
chapter 8	The Impact of Criteria Correlation on Multicriteria
	Selection
	8.1 Introduction
	8.2 Evaluation of Criteria Interrelationship
	8.3 Criteria Correlation and Profitability of Pareto
	Selection
	8.4 Criteria Correlation and Profitability of
	Selection Using the Convolution Method 212
	8.5 Summary
	Conclusion
	Bibliography
appendix	Basic Notions
	Index

This page intentionally left blank

Acknowledgments

The authors would like to express their gratitude to the team at High Technology Invest Inc. Special thanks are due to Arsen Balishyan, Ph.D., and Vladislav Leonov, Ph.D., for their skillful assistance in research and manuscript preparation. We are also indebted to Mikhail Kolkovsky, Sergey Anikeev, and Eugen Masherov, Ph.D., for their useful comments and continued help at all stages of this project.

About the Authors

Sergey Izraylevich, Ph.D., chairman of the board of High Technology Invest Inc., began his career as a lecturer at The Hebrew University of Jerusalem and Tel-Hay Academic College. He received numerous awards for academic excellence, including Golda Meir's prize and the Max Shlomiok honor award of distinction. Sergey has traded options for more than 10 years and engages in creating automated systems for the algorithmic trading of options. He is the author of numerous articles published in highly rated, peer-reviewed scientific journals. Sergey is a columnist for *Futures* magazine.

Vadim Tsudikman, president of High Technology Invest Inc., is a financial consultant and investment advisor specializing in derivatives valuation, hedging, and capital allocation in extreme market environments. With 12 years of options trading experience, he develops complex trading systems based on multicriteria analyses and genetic optimization algorithms. In co-authorship with Sergey Izraylevich, Vadim contributes articles to *Futures* magazine on the cutting-edge issues related to options pricing, volatility, and risk management.

Introduction

What the Book Is About and Who Should Read It

This book discusses the procedures of multidimensional search, selection, and utilization of potential trading opportunities existing in the options market. It contains no magic rules promising quick and guaranteed enrichment. Instead, you find comprehensive research aimed at discovery and practical application of statistical regularities and probabilistic characteristics of option trading. The aim of our systematic approach is not the creation of ever-winning strategies. Rather we strive to implement a realistic idea—developing a system of algorithms and rules that provide you with statistical advantage over the average market participant. The trading system based on consistent application of the principles discussed in this book enables you to create and maintain positions with high (higher than the market average) expected profits and lower forecast risks.

The substantial part of this book is devoted to the problem of selection. Statistical edge and probabilistic advantages depend on our ability to select the best variants from a great number of available alternatives. The options market is incredibly broad and diverse, whereas promising trading opportunities are rare and hard to identify. To avoid missing the chance to discover these scarce "pearls," an ample quantity of alternatives should be thoroughly estimated and analyzed. Continuous analyses of large data sets covering the entire options market is the only way to identify sparse trading opportunities that can be described as "the best of the available ones." Therefore, the issues related to the development, optimization, and practical application of selection criteria are discussed broadly and examined in depth throughout the book.

This book is intended for you—traders, investors, portfolio managers, theoreticians, and economists—with different grounding level in options and mathematics.

If you are familiar with the basics of statistics and probability theory and have mastered the fundamentals of option trading, you can now proceed to reading this book. For those of you who have no previous experience with options but are familiar with the first two disciplines, we recommend you to start with the appendix in which we list the main definitions and explain the notions and terms that are necessary to understand the contents of the book.

Those who are not familiar with probability theory and statistics have two options. You can start reading without delving into proofs and arguments, focusing rather on patterns and regularities described in the text and on conclusions resulting from them. In this case you must rely on the results presented by the authors and fully trust the validity of their judgments and conclusions. An alternative way, which is to dig into the basics of statistics and probability

theory, can enable you to examine the material of the book critically. Even superficial knowledge of the basics of these subjects provides an opportunity to form your own opinion on many important issues of option trading. The first way can take less of your time and effort, whereas the second one allows for getting the most out of this book.

Introduction

History of options dates back thousands of years. Social and commercial relations governed by rules similar to option terms came into existence at the dawn of human society. Various records are found in ancient documents and archeological sources dating back to the ages of Pentateuch. In Genesis Jacob purchased an option to marry Laban's daughter Rachel in exchange for 7 years of labor. His prospective father-in-law, however, reneged, perhaps making this the first precedent of option default. Laban required Jacob to marry his older daughter Leah. Jacob obeyed his will, but because he loved Rachel, he purchased another option requiring 7 more years of labor. He exercised the second option on the expiration date and finally married his sweetheart.

Before the early 1970s the options market was poorly organized. Most transactions were executed over the counter, often through the mediation of banks or other financial institutions. Essential terms of trade were not standardized, and in each case they were established through negotiations of the parties concerned. There was no formal and objective pricing mechanism that could be used as the starting point to determine the option premium. The watershed point happened in 1973, when two events brought about a fundamental change in the financial world. This was the year when Fischer Black and Myron Scholes published their famous option pricing model (Black & Scholes, 1973) and the Chicago Board Options Exchange (CBOE) began trading standardized option contracts. The first event provided traders with a formalized algorithm of option pricing. Despite numerous drawbacks, this pricing model had one indisputable advantage: It enabled the comparison of market prices with a benchmark value. The second event initiated the development of an organized options market. This process is still underway today involving a growing number of investors and financial flows in option trading.

At the dawn of the new millennium, an important milestone in the development of the world derivatives market was passed. For the first time the volume of exchange traded options (having less than 30 years of history) exceeded the volume of futures traded since 1848. Since then, options have been continuously dominating among other derivatives.

The undisputed leader in option trading is the U.S. market that absorbs more than twothirds of the world trading volume. An important peculiarity of the U.S. options market is the competition of many exchanges offering the same product (that is, options for the same underlying asset). Although CBOE surpasses other exchanges with regard to the volume of traded contracts (approximately 30% of the total option trading volume), none of them Introduction

controls more than one-third of the market. Such a competitive environment contributes to liquidity growth, spread shortening, and commission declining that attracts new market participants.

The prospects of options market development are beyond any doubt. Every year brings in additional financial flows; new trading strategies evolve; and option-based advanced structured products become more and more popular. As time goes by, the influence of large institutional investors will strengthen. In the last several years hedge funds became one of the dominating market drivers, and analysts forecast further inflow of their capital into the option trading. At the same time activity of individual investors on the derivatives markets is expected to become more intense.

The area of options application is extremely wide. Mutual funds, banks, and investment companies use options as an instrument to regulate their investment risks. Buying Put options prevents financial institutions from liquidating long positions when they anticipate the underlying asset plunging. On the other hand, when market growth is forecast, buying Call options limits potential losses (if the forecast fails) to the premium paid for options. Buying options also creates considerable leverage adding to the investment potential and increasing the effectiveness of asset management.

Producers of various goods and consumers of raw materials use options to hedge the risks of market price fluctuations. For example, by purchasing a Put option, an oil producer ensures that its future output will be sold at a price not lower than the option strike price. This is the way the company can be secured against a possible fall in the price of its production. On the other hand, an oil-refining company can buy a Call option for oil, thereby ensuring that its raw material will be purchased at a price not higher than the option strike price. Thus, the oil consumer can be secured against the price growth. International companies can hedge currency risks of their export/import operations by purchasing corresponding currency options.

Use of options to manage risks is called *hedging*. Another area of applying options, often opposed to hedging, includes a class of speculative strategies aimed at earning profits by creating various structures composed of long and short options.

Speculative option strategies give investors broad opportunities incomparable with possibilities provided by other financial instruments in respect to their flexibility and potential profitability. The main feature of options distinguishing them from the majority of other financial instruments is the nonlinearity of their payoff function (which is the relationship between profit and the future underlying asset price). This feature enables the creation of option combinations possessing almost any desired profit profile that makes options an indispensable instrument in achieving various goals for many financial market participants.

This book is intended for investors who strive to make profits using speculative option trading. *The principles described here can be applied to all option strategies.*

Although some of them are frequently used to demonstrate the techniques of discovering the trading opportunities, whereas other strategies are not even mentioned, this selectivity is merely due to our wish to keep the text within reasonable limits.

The systematic approach presented in this book is based on universal principles that can be applied to options on any type of underlying assets: stocks, futures, currencies, interest rates, and commodities. The same is true regarding different markets: Despite certain national specificities in legislation and regulation terms, options markets of all countries are suited for implementing the systematic approach.

To illustrate the different aspects of the systematic approach and to demonstrate its potential effectiveness in exploring the opportunities of option trading, we use historical data from U.S. exchanges. *The research described in this book is based on a database containing 7 years of price history of 2,500 stocks and their options.*

Options: What Is Known and What Is Not

In the 1980s, when the first option exchange and the first pricing model emerged, options markets began to develop so fast that the existing theoretical background could not satisfy increasing practical needs. The facilities required to store and to process information incoming from trading floors were not yet established. As a result, statistical data processing and theoretical developments could not satisfy growing demands of market professionals.

However, as time goes by the stream of information grows and the scope of theoretical research widens. Every year brings more and more professional publications on the subject. Options are thoroughly studied at universities, becoming one of the most popular topics of economical, mathematical, and interdisciplinary research. Option exchanges arrange seminars popularizing basic knowledge among beginners and organize advanced-level courses intended for market professionals.

A significant bulk of knowledge on options has been accumulated. These attainments are systematized and published in popular and professional sources. The literature on options can be divided into two main categories.

The first category includes theoretical research on the basis of financial mathematics. A substantial number of scientific articles and books are devoted to the development of advanced option pricing models. They apply probability theory and discuss various complicated issues, including volatility abnormalities, nonlinearities, and interrelationships between parameters.

A strict and extensive mathematical background of option theory is given by Peter James (James, 2003). This book represents the basis for researchers entering the world of options, though its complexity makes it comprehensible only to specialists with deep knowledge of mathematics. The basics of derivatives theory are perfectly described by John Hull (Hull,

2008). This is a textbook that covers all essential issues from basic terminology to complicated problems of financial mathematics. Option pricing is widely discussed in Espen Haug's book (Haug, 2006). It can be used as a universal handbook covering up-to-date progress in price modeling (see also Achdou and Pironneau, 2005, Rouah and Vainberg, 2007). Mathematical fundamentals of derivatives theory (not only options) are widely covered by Salih Neftci (Neftci, 2000). Although the majority of theoretical works have not yet been implemented, some of the mathematical models are widely used by option exchanges, brokers, market-makers, and traders. The ability to apply theoretical attainments becomes increasingly essential and publications dedicated to this issue gain considerable practical value (Reehl, 2007).

Various aspects of volatility modeling and their implications on derivatives pricing were reviewed by Jim Gatheral and Nassim Taleb (Gatheral and Taleb, 2006). The authors examine all main properties of stochastic, local, and implied volatilities and describe many classical and advanced mathematical models. A special emphasis is placed on the dynamic properties of the volatility surface and its relationship to options valuation. The discussion of volatility derivatives, barrier end exotic options is of particular interest. Besides this work, the theoretical problems of volatility modeling and forecasting were comprehensively treated by Ser-Huang Poon and Riccardo Rebonato (Poon, 2005, and Rebonato, 2004).

The second category of publications is based on practical option trading and summarizes the experience accumulated by market practitioners. It discusses strategies based on combining different options and describes methods of building desirable profit profiles on the basis of option positions structuring (Banks and Siegel, 2007, Cohen, 2005, Cohen, 2009, Courtney, 2008, Vine, 2005). Strong emphasis is placed on methods of deriving arbitrage profit.

Lawrence McMillan is a widely known author of popular books on options. His publications (McMillan, 2002; Lehman and McMillan, 2003) include a detailed description of different option strategies and are extremely useful. The author highlights a multitude of versatile techniques indispensable for any option trader. Plentiful examples based on real market data, simple language, and broad coverage—those are the distinguishing features of his books. You can find not only an encyclopedic review of option strategies in McMillan's books, but also a comprehensive description of delta-neutral hedging, arbitrage, and other specific techniques.

An excellent example of a handbook covering most aspects of option trading is the work by Michael Thomsett (Thomsett, 2009). An option trader can find there a detailed description of many useful strategies. The problems of return calculation and risk evaluation are also discussed in detail. The author gives much attention to technical aspects of option trading—information sources, taxation, accounting for dividends, and so on.

Books in which authors do not limit themselves to mere review of option strategies but discuss serious theoretical and practical issues without involving complicated mathematics are also helpful.

Sheldon Natenberg (Natenberg, 1994, 2007) describes the key elements of option theory in a popular and yet precise language. He discusses the peculiarities of implied volatility behavior and investigates the characteristics of the Greeks and specifics of their application as the instruments of risk analysis. Without superfluous mathematics, the author investigates such important phenomena as volatility smiles and skews. Comprehension of complicated theoretical issues is facilitated by intelligible charts and tables.

The book by Allen Baird (Baird, 1992) targets the same audience. Being a fairly comprehensive introduction to option theory and practice, it spares the reader the wilds of complicated mathematics. Accurate description of risk management basics is among the main vantages of the book. The section devoted to the most typical mistakes made by trading beginners also deserves a special mention.

Certain books are dedicated to specific option strategies. For example, the idea of volatility trading is popularly described in the work of Kevin Connolly (Connolly, 1997). Without resorting to complicated mathematics, the author applies dynamic hedging to combinations consisting of options and their underlying assets.

The book by Nassim Taleb (Taleb, 1997) also discusses various aspects of dynamic hedging and peculiarities of delta-neutral volatility trading strategies. This is the work written by a professional with years of experience in risk management. Although containing some inevitable portion of mathematics, it is still comprehensible to the majority of readers. In most cases the author uses diagrams and tables instead of formulae.

The book by Leonard Yates (Yates, 2003) belongs to the same category. The author discusses interesting ideas and gives ground for original trading strategies based, in particular, on negative correlation between VIX and S&P indices. The strategy is tested using historical data and the results indicate its potential applicability.

Many particular features of options trading were recently covered in impressive depth. These include pricing and risks associated with exotic options (De Weert, 2008), application of foreign exchange (Wystup, 2007), and commodity options (Garner & Brittain, 2007), trading at expiration (Augen, 2009), intraday trading (Augen, 2009), protective strategies based on Put options, and so on.

Our knowledge on options goes beyond the literature dedicated to this narrow topic. Theory and practice of option trading apply various elaborations originating from different areas of finance, statistics, probability theory, and applied mathematics. For example, creating their classical option pricing model, Black and Scholes used the well-known lognormal distribution that was widely discussed and cited in statistical and mathematical literature. Later other authors created their own pricing models using other known distributions.

The potential benefit of adopting ideas from adjacent scientific fields is far from being exhausted. For example, in classical option pricing theory the assumption of randomness of underlying asset price changes is the most questionable issue. Basically, it follows from applying lognormal distribution and means that the underlying asset price moves according Introduction

to geometrical Brownian motion laws. The work of Edgar Peters (Peters, 1996) represents an interesting example of a more sophisticated approach to the description of price behavior. It applies chaos mathematics, fractal theory, and nonlinear dynamics to account for asset price fluctuations. Peters claims that these models describe price behavior more accurately than standard probability distribution functions. Therefore, their application opens the gates for more accurate option price modeling. There is a lot of work to be done here, and new research of physicists and mathematicians will surely contribute to elaborating option theory.

The up-to-date achievements in the sphere of options theory can be summarized as follows. There is an adequate, albeit with certain drawbacks, option pricing model. Numerous versions of the basic model, eliminating some of its drawbacks and making the estimations more accurate, are also available. The basic principles of creating option pricing models, based on assumptions about the main underlying asset characteristics, are reliably established. Basic option risk indicators ("the Greeks") are grasped. We know their features, interrelationships, and applicability in different situations. Various aspects of implied volatility behavior, including its dynamics, specific relationships with different parameters, and numerous anomalies, are profoundly investigated. We also possess an extensive set of advanced option strategies allowing construction of almost any desired payoff profile.

Despite this impressive progress, some important aspects still remain beyond theoretical and practical studies. Next we summarize issues still requiring additional investigation.

The main topic of theoretical research (though directly related to investment practice) is the determination of the fair option value. The term *fair value* stands for the price that implies zero profit for both option sellers and buyers. This requires creation of realistic option pricing models (Katz and McCormick, 2005). It is common knowledge that apart from parameters that are objectively defined (current underlying asset price, strike, risk-free interest rate, and so on), the option price is determined by the forecast of underlying asset price dynamics. In the classical model this forecast is expressed by a probability density function of lognormal distribution that is specified by two parameters: variance derived from historical volatility and mean value that is usually considered to be equal to the current price. This form of forecast has a number of drawbacks, though attempts to use other probability distributions gave only local improvements and added new drawbacks. Hence the main gap in option theory can be defined as the absence of alternative methods for creating probability forecasts of the future underlying asset price.

If the price is assumed to be a continuous value, then the forecast can take the form of probability density function. The construction of such functions should be the principal topic of future research. We consider attempts to create one universal function for all cases to be unproductive. It should rather be a set of rules and algorithms for generating a whole class of density functions, each of which will be appropriate in certain conditions. The development of effective algorithms generating appropriate probability density functions will minimize the difference between modeled prices and fair values of options.

Apart from developing high-quality probabilistic forecasts, further research should target the development of optimization algorithms for parameters used in option pricing models. Even in the Black-Scholes model—which is relatively simple and contains only a few parameters—the outcome strongly depends on the variance value. Historical volatility, which is usually used to derive variance, depends on the length of the historical period used for its calculation. The value of this parameter can change the modeled option price considerably. As models become more complicated, the number of parameters increases and their combined influence becomes more pronounced.

Another essential drawback of option theory consists in the insufficient development of specific risk indicators. (Some alternative indicators are described in Izraylevich and Tsudikman, 2009d, 2010.) The majority of works on this issue are based on calculating the Greeks that are derivatives of the option price with respect to the underlying asset price (delta), volatility (vega), time (theta), and the interest rate (rho). (Derivatives of higher orders are also used.) Derivatives are calculated analytically using formulae of option pricing models. This implies that risk indicators obtained in this way inherit all the drawbacks of initial models. Such an approach to expressing option risks seems to be rather lopsided. Just as options market prices rarely match with the modeled ones, the Greeks calculated analytically almost never coincide with real changes in option values. We believe that future research of option risk management should focus on three main issues.

The first one relates directly to the problem of improvements in pricing models. The modeling formulae should be modified to include not only high-quality probabilistic forecasts (previously mentioned) but also to enable calculation of useful indicators (derivatives or any other coefficients) that accurately reflect corresponding risks.

The second issue represents the empirical study of option price increments in response to changes of underlying asset price, volatility, time to expiration, and risk-free interest rate. The patterns established in the course of these investigations can then be used (i) as independent risk indicators, (ii) for adjustment of the Greeks derived analytically, and (iii) to calibrate option pricing models.

The third issue corresponds to the estimation of risks of an option portfolio as a whole entity. Some risk indicators, such as theta and rho, are additive. Hence the dependence of the portfolio on time decay or interest rate change can be easily expressed as the sum of thetas or rhos of all options included in the portfolio. On the contrary, delta and vega are nonadditive. Therefore, if the portfolio consists of options on several underlying assets, summing separate deltas and vegas is meaningless. One of the possible ways to solve this problem is to present the delta of each option as a derivative with respect to some index (such as S&P500 or NASDAQ) rather than with respect to the price of a corresponding underlying asset. (This issue is discussed in Izraylevich and Tsudikman, 2009b.) In the same way vegas of separate options can be expressed as derivatives with respect to volatility index (such as VIX or VXN) rather than with respect to volatility of a separate underlying asset.

These procedures produce additive deltas and vegas that enable calculation of risk indicators (by summation of additive deltas and vegas) characterizing the whole portfolio. Other ways to estimate risks of a complex portfolio should also be examined. Research in this field will certainly bring useful practical results.

In this review we outlined what is already known about options and how much still lies ahead of us. We defined the main lines of future research that are, in our point of view, of special interest. Some of the gaps in option knowledge are partially filled in this book.

The Concept of the Systematic Approach

This section introduces the basic concept of the systematic approach including its philosophy, objectives, and methodology. Here we overview the essence of operations required for consecutive execution of valuation, comparative analyses, and selection procedures. We strongly recommend you get acquainted with this material as it represents an allembracing description of the general framework for systematic options trading.

The Goals and Objectives

One of the main issues in the option trading is the problem of selecting the best variants among many available alternatives. The choice is wide and the objects to examine and assess are compound structures. Although continuous functioning in such complicated environment hampers the investment process significantly, it provides at the same time a broad spectrum of promising trading opportunities.

In the literature and in multiple services offered by brokerage firms and Internet sources, the problem of choice is generally solved through application of different market scanners and rankers. A typical scanner screens the market for underlying assets that currently have extreme characteristics, such as divergence between historical and implied volatilities, daily volatility fluctuations, changes in trading volume, and so on. Afterward a ranker orders underlying assets according to the suitability for a particular option strategy. Then suitable combinations should be designed for all chosen underlying assets. Because a great number of combinations can be constructed for a given underlying asset within a given strategy, it is usually advised to use combinations' payoff charts (the functional relationship between the price of the underlying asset and a combination's profit estimated for a certain future date) as a basis for their comparison and decision making. However, in most cases visual analysis is quite unfeasible if a large quantity of option strategies and underlying assets have to be compared simultaneously.

We regard the choice of suitable underlying assets for the *a priori* defined strategy as a differential approach. It is a forced measure resulting from the imperfection of analytical tools limited to simple scanning and visual analysis of payoff functions. Differential selection

deprives the investor of the potential to utilize the whole spectrum of various trading opportunities provided by the market completely and effectively.

What we oppose to a differential approach is an integral systematic approach based on the strictly formalized assessment criteria, universal procedures of multicriteria analysis, and well-structured selection algorithms. The systematic approach enables simultaneous processing of a considerable number of option strategies and underlying assets. Without such an integral system, the investor has little or even no chance to make prompt selection decisions and to adapt successfully to changing market conditions.

The main goal of the systematic approach is to create a complex portfolio containing a potentially unlimited number of option combinations corresponding to a variety of strategies and underlying assets. Its application ensures that all trading opportunities appearing at any particular moment will be thoroughly estimated and none of the variants worth considering will be omitted. The systematic approach is absolutely indispensable for turning option trading into a long-term continuous process of income generation with controllable parameters of risk and profitability.

Valuation, Comparative Analysis, and Selection

The systematic approach is realized through consecutive execution of the following procedures: valuation, comparative analyses, and selection. These procedures are applied to the multitude of option combinations. The combination represents a complex structure consisting of any number of long and/or short individual options corresponding to certain underlying assets. Each option combination can be characterized by the shape of its payoff function. When referring to the *option trading strategy*, we will imply a certain definitive shape of the payoff function that is inherent to all combinations belonging to the same strategy and that is qualitatively distinguishable from payoff functions of combinations not belonging to this strategy. The set of option combinations available at any given moment in time for valuation, analyses, and selection of promising trading opportunities will be referred to as the *initial set*.

Valuation

Option combinations are valuated through the application of strictly formalized criteria developed specially for this purpose and expressing potential profitability and risk of the assessed variants in different ways. Criteria represent mathematical constructions with different degrees of complexity and one or many parameters. Optimization of parameters is performed either by means of statistical analyses of historical time series or by expert forecasts. Because parameters optimized on historical data are inclined to suffer from the

disadvantage of curve fitting, close attention should be paid to the validity of statistical patterns used to determine their optimal values. Expert forecasts also have significant drawbacks because they reflect opinions of particular specialists and thus represent rather subjective estimates. A systematic approach, applying both statistical analyses and expert forecasts, allows diminishing their drawbacks while amplifying advantages of these two parameterization methods.

Development of sophisticated criteria capable of valuating option combinations adequately, and optimization of their parameters, are the crucial issues that determine the practical success of systematic approach. The first part of this book discusses the basic principles of criteria construction and parameterization; the main criteria are described and analyzed in detail.

After being valuated by criteria, every combination receives a numerical characteristic reflecting its investment attractiveness. Option combinations can be valuated by one or several criteria. In the latter case the number of characteristics attributed to each combination is equal to the number of criteria.

Comparative Analysis

Following the completion of the valuation stage, the characteristics attributed to combinations need to be analyzed. During the analysis every combination is compared with all the others according to their characteristics. As a result, all variants constituting the initial set are ordered according to their quality indicators.

If the valuation was based on several criteria, then the analysis generates several orderings. In this case the same combination can have different positions in different orderings. For example, a combination can be the best one according to its expected profit, but at the same time it can be at the end of the list in an ordering obtained by the application of some risk-related criterion. Subsequently all orderings can be either used separately or combined into a unified one.

The unified ordering can be either complete or partial. Usually partial ordering appears when a complete one is unachievable. This may happen if some items turn out to be incomparable by certain criteria or if they are valued differently according to different criteria. In such cases the entire set of alternatives is divided into groups, and these groups are consequently ordered as joint entities. Different methods appropriate for execution of such procedures are discussed in Chapter 7, "Basic Concepts of Multicriteria Selection as Applied to Option Combinations."

Selection

At the next stage the results of the comparative analysis are used to select a limited number of combinations possessing superior quality characteristics. This procedure needs to be arranged thoroughly because it leads to the irreversible decision as to which combinations will enter the portfolio and which ones will be rejected.

You need to consider three main principles when choosing combinations suitable for inclusion into the portfolio.

- The number of combinations selected should be large enough to maintain diversification of the portfolio above some minimum level. Like in the classic portfolio theory, it minimizes specific risks related to individual underlying assets.
- Criteria values of selected combinations should exceed the values of the rejected ones. The minimal threshold for this excess should be established for each particular situation.
- The relative superiority of some combinations over others (resulting from the comparison of their criteria values) should not be considered as the sufficient reason for selection. The absolute criteria values must also be taken into account. For example, between two combinations with an expected return of -\$2 and -\$10, the first variant is preferable and, in principle, can be selected as the one with relatively better characteristics. However, the absolute value of the expected return corresponding to the first combination is negative and hence this combination, just as the second one, cannot be selected to enter the portfolio.

In practice, however, these principles contradict one another. Thus, following the second and the third principles an investor endeavors to decrease the number of combinations selected. At the same time the principle of portfolio diversification induces the opposite tendency—to increase this number. Thereby the structure of the resulting portfolio represents a compromise (trade-off) between all three principles.

The selection procedure represents a set of rules determining how to draw the line separating potentially profitable combinations from those that lack such potential. Consider a simple situation: The initial set consists of N combinations ordered according to the values of a certain criterion; the investor must select N' best variants out of N alternatives. This problem may be solved by creating one or several utility functions. The argument of such function is the number of selected combinations (numerical value of the place occupied by the last selected combination in the ordering). The value of the utility function is an indicator reflecting the measure of utility function may be defined as the relationship between an average return (the maximum drawdown, the Sharpe ratio, or any other characteristic reflecting the investor's satisfaction) and the number of combinations selected.

Analytical methods are not applicable to the majority of utility functions because no formulae establish the relations between the value and the argument of these functions. Hence the values of utility functions are usually derived empirically from historical time series using different statistical techniques.

If several utility functions are used simultaneously, they need to be combined into one unified function. Such unification is possible because all utility functions have the same argument (the number of selected combinations). The main requirement for the methods used to combine different utility functions is the unambiguity of the outcome that must be consistently interpretable. It means that the resultant function should be unimodal with a single evident maximum corresponding to the optimal number of combinations to be selected. Statistics offers several methods to combine empirical functions; the most popular are multiplicative and additive convolutions. We have developed an additional method—a minimax convolution (see Chapter 5, "Selection of Option Strategies") that in most cases brings more reliable and unambiguous results.

Sequence of Operations, Notion of a "Matrix" and Its Reduction

The initial set consists of a huge number of option combinations that must be processed during the execution of valuation, analyses, and selection procedures. Suppose that at any time moment there are m_i options traded for every underlying asset *i*. Assuming that any option can either be absent or present in the combination (in the latter case it can be either long or short) and that the proportion of different options is the same in all combinations, the number of possible combinations for one underlying asset is determined as 3^{m_i} . Accordingly, the total number of combinations for *n* underlying assets can be estimated as follows:

combinations =
$$\sum_{i=1}^{n} 3^{m_i}$$

Even if only 1,000 underlying assets are available for trading, and on average only 20 different options are traded for every underlying asset, then the procedures of valuation, analyses, and selection will cover more than 300 billion combinations! Moreover, the possibility to use unequal proportions of different options within one combination—which is quite realistic—generates a truly enormous number of variants to process. This number is so gigantic that computational procedures become unrealizable even for the most advanced computer hardware. Therefore, the initial set needs to be decreased to some reasonable quantities that are possible to work with on personal computers. You can achieve this decrease through the creation of combination-generating algorithms that produce only

potentially appropriate combinations instead of generating all possible variants. (Their appropriateness is determined by specific requirements and limitations of the particular investor.)

First, the investor must decide what strategies to use and then generate combinations corresponding exclusively to these strategies. This can significantly decrease the number of variants in the initial set. Then additional reasonable limitations should be applied within every strategy. For example, the following limits can be used for the *short strangle strategy*: The strike of the Call option must be greater than the strike of the Put option; the difference between Call and Put strikes must not exceed 25% of the underlying asset's price; the ratio of Put options to Call options must be between 0.8 and 1.2. Such limits, on the one hand, are well founded and, on the other hand, they do not prevent an investor from taking full advantage of the majority of opportunities appearing in the options market. At the same time, these limitations reduce the initial set to such an extent that makes it processible for personal computers.

Further facilitation of computational procedures can be achieved if selection is realized as a series of consecutive subselections. We propose to adhere strictly to the following sequence of operations. Before initiating any selection procedure, a range of potentially suitable underlying assets and trading strategies should be determined. At the same time the algorithms used to generate option combinations must be established. After that the initial set of variants available for trading can be represented as a three-dimensional space ({underlying assets × strategies × combinations}) on which the consecutive subselection procedures are executed.

If the algorithms used to generate option combinations allow creating only one combination for every underlying asset within every strategy (that is, one single combination corresponds to each {underlying assets \times strategies}), then the three-dimensional space of the initial set turns into a two-dimensional space. The two-dimensional initial set can be visualized as a table with lines corresponding to underlying assets and columns—to strategies. Each cell of this table contains one option combination relating to a given underlying asset and to a certain strategy. Such a table can further be referred to as a *two-dimensional matrix*.

If several option combinations are created for every underlying asset within every strategy, then each cell in the table will contain more than one combination for any {underlying assets \times strategies}. In this case the two-dimensional matrix becomes a three-dimensional one. (Its elements form the initial set.)

The procedure of selection can be viewed as a *reduction of the three-dimensional matrix*. We propose to realize it as a sequence of three consecutive operations (each of which can be regarded as a subselection).

The first operation represents selection of one or several best option combinations corresponding to a specific underlying asset and a given strategy. Suppose that the initial set consists of 30,000 elements (1,000 underlying assets, 5 strategies, and 6 combinations for every {underlying assets \times strategies}). If during the first subselection procedure only one combination is chosen out of the 6 possibilities, then the initial set of 30,000 items decreases to 5,000 and the three-dimensional matrix becomes two-dimensional. Chapter 4, "Selection of Option Combinations," discusses the methodology of this operation.

The second operation consists of choosing one or several superior option trading strategies for every underlying asset. In our example this leads to a further decrease of the initial set. If only one strategy is chosen, the initial set declines to 1,000 combinations. The result of this operation is the ultimate reduction of the matrix because after the execution of this subselection a unique list of underlying assets corresponds to every strategy. Therefore, the remaining part of the initial set cannot anymore be presented as an entire table without gaps. This operation is discussed in Chapter 5.

The third operation is intended to select the best variants from the lists of underlying assets corresponding to every strategy. If this procedure selects approximately 10% of combinations out of those that were chosen during two previous operations (we assume this percentage as an average estimate though in practice it can vary substantially), then the initial set is finally reduced to just 100 variants. Chapter 6, "Selection of Underlying Assets," describes this operation in detail.

This sequence of operations represents only one possible way to perform the procedures of valuation, analyses, and selection. Some other, more complicated approaches to the reduction of the initial set matrix can be developed. However, the scope of this book is limited to the preceding scheme because even such a relatively simple algorithm has more than enough particular features and specific peculiarities.

Overview of Trading Opportunities

This book covers various aspects of dealing with trading opportunities provided by options: from their detection and investigation to selection and deriving profits. This statement *a priori* assumes the existence of trading opportunities. Although this is quite obvious to the authors, it would be more appropriate to demonstrate the permanent presence of various trading opportunities in the options market. Besides, it is also useful to investigate their dynamics and structure.

The aim of this section is to perform the statistical investigation of trading opportunities existing at different moments in time for various underlying assets. An overview of trading opportunities is also expedient because it provides you with analytical tools for evaluating the potential profitability of different options markets and underlying asset types.

What Is a Trading Opportunity?

A *trading opportunity* is the deviation of the market price of an option (or any option combination) from its fair value. The *fair value* is the price that implies zero profit for both the seller and the buyer of the option. This interpretation is related to the efficient market hypothesis stating that any new information is immediately priced in and hence all traded assets are fairly valued, and extracting profit is impossible neither for sellers nor for buyers. It is common knowledge that the efficient market concept is an idealization unachievable at present days. Financial markets are ineffective; asset prices constantly fluctuate and deviate from their fair value thereby creating various trading opportunities.

Because we define the trading opportunity as the difference between the market price and the fair value, we need to establish the algorithms to estimate these variables.

At each moment *the market price* is characterized by three indicators: last, bid, and ask prices. The first indicator is of little importance because it deals with the past whereas we are interested in current trading opportunities. To discover them it is preferable to use bid and ask prices. If the investor prefers to be on the conservative side, the worst of these two prices should be used (that is, ask price—when buying options, and bid price—when selling them). This approach decreases the probability of a mistake but reduces the number of trading opportunities considerably. The less conservative investor can use the combination of bid and ask price—their simple average or weighted average with different weights for the best and the worst price. In this case the probability of erroneous inclusion of the option into the category of "trading opportunities" is higher, but the sample is more representative.

Estimating *the fair value* of any asset is an extremely difficult task, and options are not an exception. Their distinctive feature is that, in contrast to other financial instruments, options have expiration dates. This enables us to evaluate the accuracy of the fair value estimate in a reasonably short time period. The common way to estimate the fair value of options is the Black-Scholes formula and other similar models. However, they have a number of significant drawbacks and cannot be used to obtain the fair values suitable for the estimation of trading opportunity. That is why we use another method to get more accurate fair value estimates.

Method for the Evaluation of Trading Opportunities

The quantitative expression of trading opportunities can be obtained by subtracting the fair value of an option from its current market price. For comparability of results the difference should be normalized by the strike price (allowing us to express the differences between fair and market prices in percentage).

We assume the market price of an option to be equal to the average between the bid and the ask prices. The option fair value can be calculated using the method proposed by Ralph Vince (Vince, 1992):

Option fair value =
$$\sum_{i=1}^{N} (p_i a_i)$$
,

where p_i is the probability of outcome *i*, a_i is the profit or loss of outcome *i*, and *N* is the number of possible outcomes.

To obtain the most accurate estimate of the fair value, Vince permits looking into the future. Knowing the underlying asset price at the expiration date, we can find out the exact fair value of any option. In this case we have the only outcome (N=1) with probability $p_i = 1$. Accordingly, the fair value of the option is a_i .

For the Call option a_i is equal to the difference between the underlying asset price (UAP) at the expiration date and the strike price (SP) if UAP > SP, otherwise $a_i = 0$. For the Put option a_i is equal to the difference between the SP and the UAP at the expiration date if UAP < SP, otherwise $a_i = 0$. To be fully accurate, we need to discount this fair value by the risk-free interest rate normalized by the time to expiration. However, within the framework of current research, this correction can be neglected.

Zero (or close to zero) difference between the market price and the fair value indicates absence of trading opportunities. The positive difference indicates that the option is overvalued and there is a trading opportunity to sell it. Similarly, the negative difference indicates that the option is undervalued and there is a trading strike price opportunity to buy it.

To avoid zero fair values we evaluated trading opportunities of simple option combinations (straddles) rather than of separate options. The market prices and the fair value of combinations have been calculated for each of the 2,500 stocks and each date of the period from January 2, 2001, to August 16, 2007. Straddles were created using contracts with the nearest expiration date and strike prices closest to the current underlying price. Thus we obtained a table of 2,500 lines (according to the number of stocks) and 1,564 columns (the number of dates). Each cell of this table contains the value of the difference between the market price and the fair value of the combination corresponding to a certain date and to a given stock. In total, we calculated 3,910,000 values characterizing the presence or absence of trading opportunities.

Structure and Dynamics of Trading Opportunities

To demonstrate the intraday structure of strike price trading opportunities existing in the options market, we arbitrarily chose several dates (May 1, June 1, July 2, 2007) and analyzed the deviations between the market and the fair values on these days. The quantity and quality

of trading opportunities are vividly depicted in Figure I.1 with the market price of a combination on one axis and its fair value on another. Each point on the figure relates to the combination corresponding to a certain underlying asset.

On the whole we can observe a firm relation between the two indicators ($R^2 = 0.32$). Points at the line relate to combinations with no trading opportunities. Points to the north of the line represent overvalued combinations. Undervalued combinations are represented by points under the line. The scattering pattern of points in Figure I.1 indicates the presence of a considerable number of trading opportunities. Both overvalued and undervalued combinations are observed in large quantities. However, the extent to which they are over- or undervalued varies widely. Although many points are not situated exactly on the separating line, they are still very close to it, meaning that the trading opportunities in these cases are negligible.

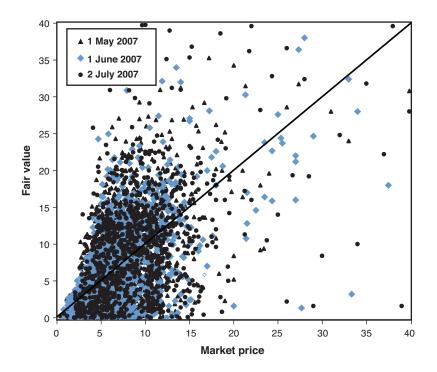


Figure I.1 Relation between market price and fair value of straddles observed as of May 1, June 1, and July 2, 2007, for 2,500 stocks. The line separates overvalued (points above it) and undervalued (points below it) combinations. All values are expressed as the percentage of the strike price.

Introduction

We propose the following heuristic rule to separate combinations with trading opportunities from others that do not possess any trading potential or only have an insignificant one. Combinations with a difference between the market price and the fair value of no more than one percent (that is, within the range of -1% to 1%) are considered as lacking trading opportunities. Combinations with a difference that is outside this range are considered as having trading opportunities. Combinations with a difference of >1% are overvalued; combinations with a difference of <-1% are undervalued. Based on this classification we can analyze data presented in Figure I.1. (Only one date is featured in the following discussion because there is no significant difference between different days.)

The best way to represent the structure of trading opportunities is to build a frequency distribution of the differences between market and fair prices (see Figure I.2). The distribution of trading opportunities existing as of July 2, 2007, is characterized by the skew toward positive values. This indicates that overvalued combinations prevailed over undervalued ones. Only 19% of combinations fall into the -1% to 1% interval that we consider to be the range with negligible trading opportunities. Fifty-two percent of combinations are overvalued and 29% are undervalued. This means that more than 80% of stocks had the potential of realizing either short- or long-straddle strategies. Short positions could be opened for more than half of the combinations, whereas long straddles turned out to be profitable in slightly less than one-third of all cases.

The frequency distribution of differences between market and fair values deviates from normal distribution considerably (see Figure I.2). Small differences corresponding to the absence of trading opportunities are more frequent than it is expected under the normal distribution. Medium differences are observed less frequently than under normal distribution. Comparison of two distributions reveals asymmetry in distribution of trading opportunities. Moderately overvalued combinations are more frequent than under normal distribution whereas moderately undervalued combinations are less frequent. However, the situation with the distribution of big differences is the contrary—highly undervalued combinations (left tail of the distribution) are more frequent than highly overvalued ones (right tail of the distribution) (see Figure I.2).

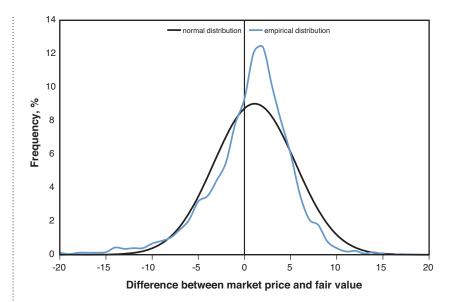


Figure 1.2 Two distributions of differences between market prices and fair values of straddles observed on July 2, 2007, for 2,500 stocks. Prices and differences are expressed as the percentage of the strike price. Positive differences correspond to overvalued straddles, negative differences—to undervalued straddles.

So far we have been analyzing the distribution of trading opportunities between different underlying assets (to be more exact, between combinations corresponding to these assets) within one day. At the next stage the time dynamics of trading opportunities will be considered for separate underlyings. We begin with one stock (AAPL will be used as an example) and calculate the difference between market prices and fair values of straddles for all dates within the period from January 2001 until August 2007. Figure I.3 shows these differences plotted against the corresponding dates. Visual analysis of these data reveals that undervalued periods alternate with overvalued periods. In general, we can say that the dynamics of this process is characterized by quasiperiodical cycles. Although at first sight these cycles have similar periodicity, their detailed investigation suggests that trading opportunities can hardly be forecasted on their basis. Introduction

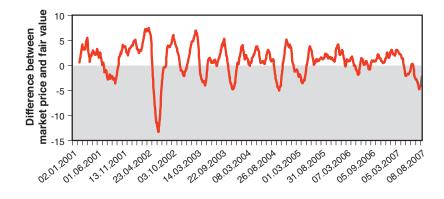


Figure I.3 Dynamics of the differences between the market price and fair value of straddles for AAPL stock. Prices and differences are expressed as the percentage of the strike price.

Using AAPL as an example we illustrated the dynamics of trading opportunities for just one underlying asset (Figure I.3). However, the analysis of other stocks (not presented here) gives similar results. The overwhelming majority of underlying assets shows similar behavior—more or less regular fluctuations between overvalued and undervalued areas. As in the AAPL case, there are periods when trading opportunities are negligibly small.

Previously we mentioned that the intensity of trading opportunities can vary widely. Although we agreed to consider the difference between the market and the fair values of more than 1% as indicating the presence of a trading opportunity, the profit potential may be quite low if the differences exceed 1% by just a slight margin. On the other hand, the difference of 5% and over has a strong profit potential. As it follows from Figure I.3 (and other research not presented here), medium trading opportunities (with a profit potential of approximately 2% to 4%) are prevailing in the market. However, it should not have a negative impact on our evaluation of trading opportunities because these medium deviations of market prices from their fair values occur quite frequently.

The dynamics of trading opportunities analyzed by the example of AAPL indicates the approximate equality of periods when the options of a certain underlying asset are overvalued and when they are undervalued. Does such uniform distribution of trading opportunities (between over- and undervalued periods) reside in all stocks? Are there underlying assets with options that are undervalued or overvalued most of the time? To answer these questions we calculated the number of days when options were overvalued and undervalued for each of the 2,500 stocks. We divided the obtained values by 1,564 (the total number of days) to express them as the percentage fraction of time. This data was used to build two distributions of time fractions: when options were overvalued and when they were undervalued (see Figure I.4).

Two distributions are shifted relative to each other: overvalued—toward the longer fractions of time, undervalued—toward the shorter fractions of time (see Figure I.4). This means that options are more often overvalued than undervalued. In general, we can conclude that most of the options are undervalued for no more than 25% to 40% of the time and are overvalued for 45% to 60% of the time (see Figure I.4). Moreover, options relating to approximately 5% of stocks are overvalued for more than 70% of the time. On the other hand, options relating to only 3% of stocks are undervalued for just 50% to 60% of the time. This means that options of some stocks are permanently overvalued, whereas constantly undervalued options are relatively rare.

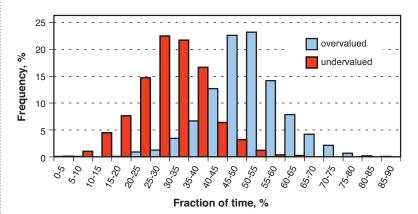


Figure I.4 Distribution of time fractions when options were undervalued and overvalued.

As shown in Figure I.1, a multitude of trading opportunities consisting of both undervalued and overvalued options exists in the market simultaneously. To get a detailed notion of their dynamic structure, we can analyze the proportions of combinations possessing trading potential and those lacking it. This investigation demonstrates how the ratio of overvalued, undervalued, and fairly valued combinations changes in time. To clarify these issues we use 2,500 straddles (one for every underlying asset) for each of the 1,564 dates. For every combination we calculate the difference between the market price and its fair value. Based on this indicator, the straddles are classified into three categories in accordance with the heuristic rule previously proposed. Combinations with a difference of >1% are considered to be overvalued; those with a difference of <-1% are undervalued. Combinations with a difference between -1% and 1% are considered to be fairly valued (that is, lacking any trading opportunities). For each date we calculate the proportion of combinations belonging to each of the three categories and observe the dynamics of their changes in time. Introduction

The proportion of fairly valued combinations is relatively stable in time—it fluctuates slightly within the range from 10% to 20% (see Figure I.5). Overvalued combinations prevail over undervalued ones. The former constitute approximately 50% to 60% throughout most of the time, whereas there are only 30% to 40% of undervalued combinations. (This corresponds with the conclusions drawn from the analysis in Figure I.5.) At the same time there are periods when the proportion of undervalued combinations rises sharply (see Figure I.5).

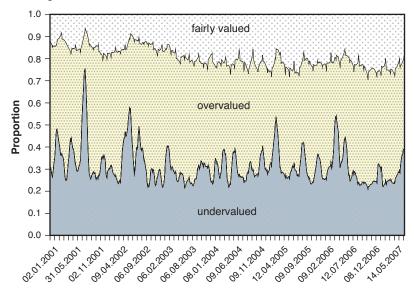


Figure I.5 Dynamics of proportions of undervalued, overvalued, and fairly valued combinations.

The following conclusions can be drawn from this statistical investigation. Considerable trading opportunities consisting of overvalued and undervalued options are constantly available in the market. However, their relative ratios have complicated time dynamics that can hardly be forecasted by discovering persistent cycles with regular periodicity. Consequently, an accurate prediction of future trading opportunities—whether most of the options will be overvalued or undervalued—seems to be unfeasible. Therefore, at every moment in time the investor should determine the trading potential of each separate combination and create corresponding strategies based on selling overvalued, buying undervalued, and excluding fairly valued options and their combinations.

Index

A

absolute price changes distribution, 58-61 absolute values of criterion, 110-112 adding combinations, 76 additive convolution, 190 algorithms of combination selection, 121-122 empirical distribution, 28-31 optimization, xxii Pareto sets, 183-189 allocation of capital, 172-173 analysis comparative, xxiv-xxv criteria effectiveness, 106-110, 150-155 dynamic, 67 factors affecting combination selection, 113-114 multicriteria, xxiv

number of combinations used in, 75-77 ranking generalized results, 140-142 methods of, 123-131 results of, 131-138 regression, 83 selection of underlying assets, 162-165 Sharpe ratios, 101 visual, xxxiv applying convolution methods, 212-214 lognormal distributions, 42 regression models to profit and value relationships, 65 area ratios, 91-95 assets fair value, xxx price variability, 57

underlying, 112 analysis of criteria effectiveness, 162-165 long-term evaluation of criteria effectiveness, 166-168 optimization, 168-176 selection of, 161-162 assignment of distribution of probabilities, 5 averaging four (4)-days data, 68 selection of periods, 72-73, 75

B

Baird, Allen, xx banks, options, xvii. *See also* options bear spreads, 233-234 behavior, prices, xxi black symbols, 149 Black, Fischer, xvi Black-Scholes model, xxx, 6, 57 break-even range, 50-53 Brownian motion, 42 bull spreads, 233-234 buy low, sell high strategy, 7 butterfly strategy, 37

С

calculations determination coefficients, 69 empirical distribution, 28-31 lognormal distribution, 15-22 ratio of expected profit and loss, 34-38 relative changes, 14 returns, xix

TimeValue, 60 unified probability density functions, 47 calendar spreads, 231-233 comparison of, 151 calls options, xvii spreads, 12 strike prices, 3 capital allocation, 172-173 Chicago Board Options Exchange (CBOE), xvi classification of criteria, 9-12 coefficients criterion effectiveness, 133, 142-145 determination, 65-69 combinations options absolute values, 110-112 analysis of criteria effectiveness, 106-110 long-term evaluation of criteria effectiveness, 115-118 selection of, 105 simultaneous analysis of factors affecting selection, 113-114 underlying assets, 112 selection of algorithms, 121-122 several days in analysis, 68 common uncertainty, 6 comparative analysis, xxiv-xxv of multicriteria, 191-202 comparisons of calendar spreads, 151 of heterogeneous strategies, 122

Connolly, Kevin, xx construction, unified probability density functions, 48-49 convolution, 157-159, 190-191 applying, 212, 214 minimax, 156 Pareto sets, 197-202 utility functions, 173-176 correlation coefficients, 65 between criterion and profit indexes, 86-87 between criterion and profit values, 85 multicriteria selection applying convolution methods, 212-214 evaluation of criteria interrelationships, 206-208 impact on, 205 selection profitability of Pareto selections, 208-212 between Sharpe ratios of criterion and profit, 88-91 criteria classification, 9-12 effectiveness, 63-64 analysis of, 106-110 correlation between profit and, 64-68 expressing indicators, 80-82 expressing profit, 77-80 long-term evaluation of, 115 - 118number of combinations used in analysis, 75-77

review of indicators, 84-89, 92-99, 101 selection of averaging periods, 72-75 selection of underlying assets, 162-168 transformation of indicators, 68-69 values of coefficients, 142-145 expected profit on basis of empirical distribution, 25-28 on basis of lognormal distribution, 15-22 expert distribution, 9, 38-39 set of standard distributions, 39-45 unified probability density functions, 45-49 forecasting, 8-9, 155 inefficient, 126 based on lognormal distribution, 13-15 mission fulfilled by, 6 modifications of empirical distribution, 31-34 multicriteria selection, 181-182 applying convolution methods, 212-214 comparative analysis, 191-202 convolution, 190-191 correlation, 205 evaluation of criteria interrelationships, 206-208 Pareto sets, 183-185

profitability of Pareto selections, 208-212 widening Pareto sets, 186-189 nonexpert, 39 nonuniversal, 50 break-even range, 50-53 IV/HV range, 53-57 ratio of normalized time value, 58-61 relative frequency criterion, 57-58 perfect, 124 philosophy of creation, 5 probability on basis of empirical distribution, 28 profit correlation between indexes, 86-87 correlation between Sharpe ratios and, 88-91 correlation between values, 85 ratio of expected profit to loss, 34-38 simplified calculation algorithms, 28, 31 specific (nonuniversal), 11-12 cycles, quasiperiodical, xxxiv

D

days, combining several days in analysis, 68 definitions, systematic approach, 4-5 degree of symmetry (DSYM), 115-117 delta, xxii derivatives, xxii theory, xviii determination coefficients, 67-69 development of possible future scenarios, 9 deviations in measurements, 144 distribution absolute price changes, 58-61 empirical criteria based on, 22-24 expected profit on basis, 25 - 28modifications of, 31-34 probability on basis of, 28 simplified calculation algorithms, 28, 31 expert criteria based on, 38-39 set of standard distributions, 39-45 unified probability density functions, 45-49 exponential, 43 frequency, xxxiii, 131, 136 Gaussian, 14 lognormal, xx, 41-42 description of, 13-15 expected profit on basis of, 15-22 normal, 14, 132 probabilities, 5, 19 standard, 8 of time fractions, xxxvi uniform, 40 domination of certain strategies over others, 122

dynamics analysis, 67 of trading opportunities, xxxi, xxxiii, xxxv, xxxvii of transformed effectiveness indicators, 69-71

E

effectiveness criteria, 63-64 analysis, 106-110 correlation between profit and, 64-68 expressing indicators, 80-82 expressing profit, 77-80 long-term evaluation of, 115-118 number of combinations used in analysis, 75-77 review of indicators, 84-89, 92-101 selection of averaging periods, 72-75 selection of underlying assets, 162-168 transformation of indicators, 68-69 values of coefficients, 142-145 monocriterion selection, 191-196 empirical distribution criteria based on, 22-24 expected profit on basis, 25-28 modifications of, 31-34 probability on basis of, 28 simplified calculation algorithms, 28 - 31

estimation of risk, xxii evaluation of criteria effectiveness, 63-64 correlation between profit and, 64-68 expressing indicators, 80-82 expressing profit, 77-80 long-term, 115-116, 118 number of combinations used in analysis, 75-77 review of indicators, 84-89, 92-101 selection of averaging periods, 72-75 transformation of indicators, 68-69 of criteria interrelationships, 206-208 of risk, xix of trading opportunities, xxx events, frequency, 8 expectation, 41 expected profit on the basis of empirical distribution (EPEM), 25-28, 115-117 on the basis of lognormal distribution (EPLN), 15-22, 115-117, 133-137 Pareto sets, 185-187 expert distribution criteria based on, 38-39 set of standard distributions, 39-45 unified probability density functions, 45-49 expert forecasts, 42

exponential distribution, 43 expressing effectiveness indicators, 80-82 profit, 77-80

F

F-tests, 132 fair value, xxi, xxx forecasting, xxii correlation between outcome and, 64 criteria, 155 expert, 42 future profit, 80 horizons, 27 as key elements of criterion, 8-9 profit probability, 80 universal criteria, 10 formal definition of systematic approach, 4-5 forming Pareto sets, 183-185 formulas Black-Scholes, xxx lognormal distribution, 14 frequency distribution, xxxiii, 131, 136 past events, 8 relative frequency criterion, 57-58 functions payoff, 15, 47, 226 option combinations, 228-234 separate options, 226-227 probability density, 19, 28 underlying assets, 171-176

unified probability density, 45-49 utility, 155-159 weight, 32 future scenarios, 9

G

Gatheral, Jim, xix Gaussian distribution, 14 generalized ranking analysis results, 140-142 geometrical Brownian motion, 42 goals of systematic approaches, xxiii-xxiv gray symbols, 149 Greeks, xxii, 11. See also risk

Η

Haug, Espen, xix hedging, xvii heterogeneous strategies, comparisons, 122 High FrIV(B), 58 history of options, xvi-xvii volatility, xxii, 16, 80 holding periods, 31 horizon of history, 27 building empirical distribution influences, 26 empirical distribution, 28 weight functions as substitute, 32 Hull, John, xviii

Ι

implied volatility, 57 in-the-money options, 58

indexes, correlation between criterion and profit, 86-87 indicators criteria effectiveness, 68-69 effectiveness expressing, 80-82 review of, 84-89, 92-101 profit, 64-68 utility, 169 individual uncertainty, 6 inefficient criterion, 126 interactive probability scenario builder, 39, 43 interrelationships, evaluation of criteria, 206-208 intraday structure, xxxi investment firm options, xvii. See also options IV/HV Pareto sets, 185-187 ranges, 53-57 ratios, 80

J-K-L

James, Peter, xviii

kurtosis, 8 layers, Pareto sets, 186-189, 210 lognormal distribution, xx, 41 applying, 42 description of, 13-15 expected profit on basis of, 15-17, 19-20, 22 long calendar spreads, 64, 84, 127-128, 231, 233 long history horizons, 27 long straddle strategies, 3, 84, 115, 127-128, 133, 137, 143, 228-229 long strangle strategies, xxxiii, 3, 84, 115, 229-230 long strategies averaging periods, 73-74 dynamics of transformed effectiveness indicators, 71 long-term evaluation, 1156-118, 166-168 losses, criteria based on ratio of expected profit and loss, 34-38

Μ

management, risk, xvii markets development of options, xviii price, xxxi matrices, xxvii-xxix maximum drawdowns, 169 McMillan, Lawrence, xix mean, 8, 17 MeanEmpiric(B,S), 31 MeanSymEmpiric(B,S), 34 measurements, deviations in, 144 methods convolution, 212-214 of criterion effectiveness, 145 - 150of long-term evaluation of criteria effectiveness, 115-116 of ranking analysis, 123-131 underlying assets, 163, 166 minimax convolution, 156, 173, 190 mission fulfilled by criteria, 6 models historical volatility, 17 optimization, 168-176 price, xxi

regression, 65-66 standard price distribution, 14 threshold parameters, 155-159 modes, 39 modification of empirical distribution, 31-34 monocriterion selection effectiveness, 191-192, 195-196 multicriteria analysis, xxiv selection, 181-182 comparative analysis, 191-202 convolution, 190-191 correlation, 205-214 Pareto sets, 183-189 multiplicative convolution, 190 mutual fund options, xvii. See also options

Ν

narrow trade ranges, 31 Natenberg, Sheldon, xx Neftci, Salih, xix nonexpert criteria, 39 nonforecasting universal criteria, 11 nonformalized analysis, 9 nonuniversal criteria, 50 break-even range, 50-53 IV/HV ranges, 53-57 ratio of normalized time value, 58-61relative frequency criterion, 57-58 normal distribution, 14, 132 number of combinations used in analysis, 75-77

0

Objectives of systematic approach, xxiii-xxiv Operations, sequences of, xxvii-xxix opportunities, trading, xxix-xxx evaluation, xxx structure and dynamics of, xxxi, xxxiii, xxxv, xxxvii optimization algorithms, xxii thresholds parameters, 155-156, 159 values, 145 underlying assets, 168-176 options combinations, 228 absolute values, 110, 112 analysis of criteria effectiveness, 106-110 bear spreads, 233-234 bull spreads, 233-234 calendar spreads, 231-233 long-term evaluation of criteria effectiveness. 115-118 selection of, 105 simultaneous analysis of factors affecting selection, 113-114 straddles, 228-229 strangles, 229-230 underlying assets, 112 criteria based on ratio of expected profit to loss, 34-38 empirical distribution expected profit on basis of, 25 - 28

modifications of, 31-34 probability on basis of, 28 simplified calculation algorithms, 28-31 expert distribution criteria based on, 38-39 set of standard distributions, 39-45 unified probability density functions, 45-49 history of, xvi-xvii lognormal distribution description of, 13-15 expected profit on basis of, 15 - 22markets, development of, xviii multicriteria selection, 181-182 applying convolution methods, 212-214 correlation, 205 evaluation of criteria interrelationships, 206-208 Pareto sets, 183-185 profitability of Pareto selections, 208-212 widening Pareto sets, 186-189 prices, 6 selection problem classification of criteria, 9-12 definition of, 4-5 forecasting, as key elements of criterion, 8-9 mission fulfilled by criteria, 7

philosophy of criteria creation, 5-6 tools for solving, 3-4 separate, payoff functions, 226-227 strategies, xix analysis of criterion effectiveness, 150-151, 155 generalized ranking analysis results, 140-142 methods of criterion effectiveness, 145-150 methods of ranking analysis, 123-131 results of ranking analysis, 131-138 selection of, 121-122 threshold parameters, 139-140 trading, xxvi values of criterion effectiveness coefficients, 142-145 ordering underlying assets, 171 unified, xxv out-of-the-money options, 58 outcome, forecasts, 64 overlapping of information, 155 overoptimization, 26

Р

parameters empirical distribution, 26 horizon of history, 32 lognormal distribution, 18

thresholds, 139-140 optimizing, 155-156, 159 values, 150 variance, 44 Pareto sets, 183-185 convolution, comparisons, 197-202 profits, criteria correlation and, 208-212 widening, 186-189 past events frequency, 8 patterns, scattering, xxxii payoff function, 15, 47, 226 option combinations, 228 bear spreads, 233-234 bull spreads, 233-234 calendar spreads, 231-233 straddles, 228-229 strangles, 229-230 separate options, 226-227 peculiarities, 7 Pentateuch, xvi perfect criterion, 124 periods, averaging, 72-75 Peters, Edgar, xxi philosophy of criteria creation, 5 Poon, Ser-Huang, xix position holding periods, 31 premium, risk, 6 prices behavior, xxi market, xxxi models, xxi options, 6 standard price distribution models, 14 strike, xxx, 3

probabilities density functions, 19 distribution of, 5, 19 sum of, 8 empirical distribution, 28 theory, xv ProbEmpiric(B,S), 31 problems, selection classification of criteria, 9-12 definition of, 4-5 forecasting, as key elements of criterion, 8-9 mission fulfilled by criteria, 7 philosophy of criteria creation, 5-6tools for solving, 3-4 ProbSymEmpiric(B,S), 34 profit criteria correlation between effectiveness indicators, 64-68 between indexes, 86-87 based on ratio of expected profit and loss, 34-38 between Sharpe ratios and, 88-91 between values, 85 expected on basis of empirical distribution, 25-28 on basis of lognormal distribution, 15-22 expressing, 77-80 probability on the basis of empirical distribution, 28 on the basis of lognormal distribution, 115-117

Pareto sets criteria correlation and, 208-212 selection methods, 193 proportional long strangle, 17-19 puts to call ratios, 3 options, xvii short options, 28 spreads, 12 strike prices, 3

Q-R

quasiperiodical cycles, xxxiv ranges break-even, 50-53 IV/HV, 53-57 ranking analysis methods of, 123-131 results of, 131-138 ratios areas, 91-95 of expected profit to loss, 34-38 of implied to historical volatility (IV/HV), 80, 115-117 of normalized time value, 58-61 Put to Call, 3 Sharpe, 11, 88-89, 91, 150-165 analysis, 101 Pareto sets, 188 underlying assets, 171 underlying assets, 169 Rebonato, Riccardo, xix reduction of matrices, xxvii-xxix

regression analysis, 83 models, 65-66 relationships, 135 regularities, observation of, 69 relationships between criterion and profit values, 81 between determination and the slope coefficients, 84 lognormal distribution, 18 between profit of combinations and positions, 97 regression, 135 relative changes, calculating, 14. See also yields relative frequency criterion, 57-58 results of long-term evaluation of criteria effectiveness, 116, 118 of ranking analysis, 131-142 underlying assets, 167-168 returns, calculating, xix review of effectiveness indicators, 84-89, 92-101 rho, xxii risk characteristics, 11 estimation of, xxii evaluation, xix management, xvii premium, 6

S

scales, thresholds, 153 scattering patterns, xxxii schematic representations, 123-125 Scholes, Myron, xvi selection, xxiv, xxvi-xxvii of averaging periods, 72-75 combinations of algorithms of, 121-122 multicriteria, 181-182 applying convolution methods, 212-214 comparative analysis, 191-202 convolution, 190-191 correlation, 205 evaluation of criteria interrelationships, 206-208 Pareto sets, 183-185 profitability of Pareto selections, 208-212 widening Pareto sets, 188-189 of option combinations, 105 absolute values, 110-112 analysis of criteria effectiveness, 106-110 long-term evaluation of criteria effectiveness, 115-118 simultaneous analysis of factors affecting, 113-114 underlying assets, 112 of option strategies, 121-122 analysis of criterion effectiveness, 150-155 generalized ranking analysis results, 140-142

methods of criterion effectiveness, 145-150 methods of ranking analysis, 123-131 results of ranking analysis, 131-138 threshold parameters, 139-140 values of criterion effectiveness coefficients, 142-145 problems classification of criteria, 9-12 forecasting as key elements of criterion, 8-9 formal definition of, 4-5 mission fulfilled by criteria, 6 philosophy of criteria creation, 5 tools for solving, 3-4 of underlying assets, 161-162 analysis of criteria effectiveness, 162-165 long-term evaluation of criteria effectiveness, 166-168 optimizing, 166-176 selective convolution, 190 selling straddles, 11 separate options, payoff functions, 226-227 sequences of operations, xxvii-xxix Pareto, 183-185 comparisons to

convolution, 197-202

sets

criteria correlation and profitability of, 208-212 widening, 186-189 of standard distributions, 39-45 Sharp ratio, 11 Sharpe ratios, 11, 88-91, 150, 153, 165 analysis, 101 Pareto sets, 188 underlying assets, 171 short calendar spreads, 64, 127-128, 231-233 short Put options, 28 short straddle strategies, xxxiii, 11, 127-128, 133, 137, 143 short strangle strategies, xxviii, 3, 229-230 short strategies averaging periods, 73 dynamics of transformed effectiveness indicators, 71 simplified calculation algorithms, 28-31 simultaneous analysis of factors affecting combinations selection, 113-114 skewness, 8 specific criteria, 50 break-even range, 50-53 IV/HV range, 53-57 nonuniversal, 11-12 ratio of normalized time value, 58-61 relative frequency criterion, 57-58 speculative option strategies, xvii spreads, 12

square of the correlation coefficients, 65 standard distribution, 8 Standard Error (SE), 132 standard price distribution models, 14 statistics, xv, 31 testing, 127 straddles, 3, 228-229 strangles, 229-230 strategies heterogeneous, comparisons of, 122 long-straddle, xxxiii options, xix analysis of criterion effectiveness, 150-151, 155 generalized ranking analysis results, 140-142 methods of criterion effectiveness, 145-150 methods of ranking analysis, 123-131 results of ranking analysis, 131-138 selections of, 121-122 threshold parameters, 139-140 trading, xxiv values of criterion effectiveness coefficients, 142-145 short-straddle, xxxiii short-strangle, xxviii speculative option, xvii and underlying assets, 112, 116-118 strike prices, xxx, 3

strong trends, 31 structure of trading opportunities, xxxi, xxxiii, xxxv, xxxvii sum of probabilities, 8 supply and demand, 7 symmetrization of empirical distribution, 33-34 synthetic approach to criterion effectiveness analysis, 150-151, 155 systematic approach formal definition of, 4-5 goals and objectives of, xxiii-xxiv overview of, xxiii

Τ

Taleb, Nassim, xix-xx testing F-tests, 132 statistics, 127 theories, xviii theta, xxii Thomsett, Michael, xix three-dimensional matrices, xxviii thresholds parameters, 139-140, 155-159 scales, 153 values, 140-144 optimizing, 145 parameters, 150 time fractions, distributions of, xxxvi value, 6 TimeValue, 60

tools, solving selection problems, 3-4

trading opportunities, xxix-xxx evaluation, xxx structure and dynamic of, xxxi, xxxiii, xxxv, xxxvii option strategies, xxiv selection problems classification of criteria, 9-12 definition of, 4-5 forecasting, as key elements of criterion, 8-9 mission fulfilled by criteria, 7 philosophy of criteria creation, 5-6 tools for solving, 3-4 transformation of criteria effectiveness indicators, 68-69 trends, 31 two-dimensional matrices, xxviii types of convolution, 157-159, 190 of uncertainty, 6

U

uncertainty common, 6 individual, 6 underlying assets, 112, 116-118 analysis of criteria effectiveness, 162-165 long-term evaluation of criteria effectiveness, 166-168 optimizing, 168-176 selection of, 161-162 unified ordering, xxv unified probability density functions, 45-49 uniform distribution, 40 universal criteria, 9-10 forecasting, 10 nonforecasting, 11 utility functions, 155-159, 169-176

V

validation of unified probability density functions, 48-49 valuation, xxiv-xxv values absolute of criterion, 110, 112 of criterion effectiveness coefficients, 142-145 fair, xxx mean, 17 profit, correlation between criterion and, 85 thresholds, 140-144 optimizing, 145 parameters, 150 time, 6 variance, 8 vega, xxii Vince, Ralph, xxxi visual analysis, xxxiv volatility, xxii, 41 historical, 16, 80

W-Z

weight functions, 32 widening Pareto sets, 186-189

Yates, Leonard, xx yields, 14