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The Art of Computer Programming

VOLUME 4
Constraint Satisfaction

FASCICLE 7

DONALD E. KNUTH

FREE SAMPLE CHAPTER

THE ART OF COMPUTER PROGRAMMING

VOLUME 4, FASCICLE 7

Constraint Satisfaction

DONALD E. KNUTH *Stanford University*

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See also `https://www-cs-faculty.stanford.edu/~knuth/sgb.html` for information about *The Stanford GraphBase*, including downloadable software for dealing with the graphs used in many of the examples.

And see `https://www-cs-faculty.stanford.edu/~knuth/mmix.html` for basic information about the `MMIX` computer.

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PREFACE

> *There is quite a good deal of information in the book.*
> *I regret this very much; but really it could not be helped:*
> *information appears to stew out of me naturally,*
> *like the precious ottar of roses out of the otter.*
> *Sometimes it has seemed to me that I would give worlds*
> *if I could retain my facts; but it cannot be.*
> *The more I calk up the sources, and the tighter I get, the more I leak wisdom.*
> *Therefore, I can only claim indulgence at the hands of the reader, not justification.*
>
> — MARK TWAIN, *Roughing It* (1872)

FREEDOM is a wonderful thing. But we can also be thankful for the constraints that give structure to our lives and provide a focus for our creative juices. We experience moments of great satisfaction when challenges have been met. Combinatorial and musical patterns that fit together almost magically can be supremely satisfying.

This booklet is Fascicle 7 of *The Art of Computer Programming*, Volume 4: *Combinatorial Algorithms*. As explained in the preface to Fascicle 1 of Volume 1, I'm circulating the material in this preliminary form because I know that the task of completing Volume 4 will take many years; I can't wait for people to begin reading what I've written so far and to provide valuable feedback.

To put the material in context, this lengthy fascicle contains part of a long, long chapter on combinatorial searching. Chapter 7 will eventually fill at least four volumes (namely Volumes 4A, 4B, 4C, and 4D), assuming that I'm able to remain healthy. It begins in Volume 4A with a short review of graph theory and a longer discussion of "Zeros and Ones" (Section 7.1); that volume concludes with Section 7.2.1, "Generating Basic Combinatorial Patterns," which is the first part of Section 7.2, "Generating All Possibilities." Volume 4B resumes the story with Section 7.2.2, about backtracking in general. Section 7.2.2.1 discusses a family of methods called "dancing links," for updating data structures while backtracking, and shows that such methods are particularly successful when applied to XCC problems — "exact covering with colors." That sets the scene for Section 7.2.2.2, which fills the rest of Volume 4B; it is devoted to the important problem of Boolean satisfiability, aka 'SAT'.

Now comes Volume 4C, whose first third will eventually consist of the contents of the present fascicle. Our theme, "constraint satisfaction," is Section 7.2.2.3, and it takes the concepts of Sections 7.2.2.1 and 7.2.2.2 to a higher level. A fresh look at the basic notions leads to significant improvements and many individual topics of independent interest. Once again it has been a great pleasure for me to put these stories together, and I hope that I can convey the joy of what I've learned to as many people as possible.

Dozens of examples tie the subject matter of this fascicle to numerous other parts of computer science and mathematics. We'll see, for instance, applications to scene analysis (computer vision); we'll construct fascinating instances of "graceful graphs"; we'll discuss efficient algorithms that embed one graph as a subgraph of another; we'll learn new ways to look ahead when backtracking, and we'll investigate new heuristics for guiding a search that backtracks through a massive space of possibilities; we'll also learn how to avoid backtracking altogether, when possible. We'll study new sparse-set data structures that lead to "dancing cells" — a technique that often is even better than "dancing links"! Of course there are recreational topics galore, even including some new takes on the classic problem of a knight's tour. Puzzle connoisseurs will enjoy learning more about fillomino patterns. And so on.

As usual, more than half of this book is devoted to exercises and their answers. In fact, Section 7.2.2.3 has approximately 500 exercises, some of which are routine and some of which aren't yet solved; all of them have been designed for self-study. I've tried to make it easy for a reader to navigate through this maze of problems by presenting them in an order that roughly parallels the threads of the main text.

Look, for example, at a "random" page — page 42, say, which discusses a variety of ways to translate graph coloring problems into SAT clauses. On that page you'll see that exercises 221, 226, 224, 225, 228 are mentioned. So you can guess that the main exercises about SAT encoding probably have numbers near 225. The exercises have also been carefully indexed.

My goal as always has been to strive for a good balance between theory and practice (and fun). I try to discuss the theoretical discoveries that are most relevant to people who write programs in the "real world," and to present those theories without using advanced jargon. I also strive to encapsulate the history of the subject, so that readers can understand the "human dimension" and the process of discovery.

While writing this section I also wrote hundreds of programs for my own edification. (I usually can't understand things well until I've tried to explain them to a machine.) Most of those programs were quite short, of course; but several of them are rather substantial, and possibly of interest to others. Therefore I've made a selection available by listing some of them on the following webpage:

`https://cs.stanford.edu/~knuth/programs.html`

Prototypes of the main algorithms can be found there, together with a few other programs that are mentioned in the answers to certain exercises. If you want to see a program called FOO, look for FOO on that webpage. In particular, you can download the programs SSXCC0, SSXCC, SSXCC-BINARY, SSMCC, and XCCDC; those experimental versions of Algorithms C, C^+, B, F, and S were my constant companions while writing the middle portions of this fascicle.

Interested readers who wish to do their own experiments can find data files for the benchmark examples in the following two collections:

`https://cs.stanford.edu/~knuth/programs/xcc-benchmarks.tgz`
`https://cs.stanford.edu/~knuth/programs/mcc-benchmarks.tgz`

Incidentally, the illustrations in Section 7.2.2.3 are numbered beginning with Fig. 100, because the final illustration of Volume 4B is Fig. 99. The editor has decided to treat Chapter 7 as a single unit, even though it is being split across several physical volumes.

Special thanks are due to Christian Bessière, Víctor Dalmau, Ralph Freese, Daniel Horsley, Peter Jeavons, Ciaran McCreesh, Patrick Prosser, George Sicherman, Christine Solnon, Filip Stappers, Peter Stuckey, Kokichi Sugihara, James Trimble, Udo Wermuth, Ross Willard, and Dmitriy Zhuk, for their detailed comments on my early attempts at exposition, as well as to numerous other correspondents who have contributed crucial corrections. Thanks also to Stanford's InfoLab for providing extra computer power when my workstation was inadequate. And above all, I thank my wife for her constant support and for help with Figs. 100 and 101.

I happily offer a "finder's fee" of $2.56 for each error in this draft when it is first reported to me, whether that error be typographical, technical, or historical. The same reward holds for items that I forgot to put in the index. And valuable suggestions for improvements to the text are worth 32¢ each. (Furthermore, if you find a better solution to an exercise, I'll actually do my best to give you immortal glory, by publishing your name in the eventual book:–)

Cross references to yet-unwritten material sometimes appear as '00'; this impossible value is a placeholder for the actual numbers to be supplied later.

Happy reading!

Stanford, California D. E. K.
23 October 2024

P.S.: **A note on notations.** Some formulas in this booklet use the notation '$\nu\, x$' for the "sideways sum" or "population count" function, as well as the notation 'ρx' for the "ruler" function. Those functions, and other bitwise notations, are discussed extensively in Section 7.1.3 of Volume 4A.

Other formulas use the notation $\langle xyz \rangle$ for the median function, which is discussed extensively in Section 7.1.1. Hexadecimal constants are preceded by a number sign or hash mark: `#123` means $(123)_{16}$.

Furthermore, there's a special list of 'Notational conventions' in the index.

If you run across other notations that appear strange, please look at the Index to Notations (Appendix B) at the end of Volume 4A or 4B. Volume 4C will, of course, have its own Appendix B some day.

> *The field of combinatorial algorithms is too vast*
> *to cover in a single paper or even in a single book.*
> — ROBERT ENDRE TARJAN, *SIAM Review* (1978)

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I always thought Volume 4 was a myth,
like the missing part of the Dead Sea scrolls.

— BILL GASARCH (blog post, 10 January 2008)

> *His Lady sad to see his sore constraint,*
> *Cride out, Now now Sir knight, shew what ye bee.*
> — EDMUND SPENSER, *The Faerie Queene* (1590)

> *The work under our labour grows, Luxurious by restraint.*
> — JOHN MILTON, *Paradise Lost* (1667)

> *Liberty exists in proportion to wholesome restraint.*
> — DANIEL WEBSTER (1847)

> *It is impossible to be an artist and not care for laws and limits.*
> *Art is limitation; the essence of every picture is the frame.*
> — GILBERT K. CHESTERTON, *Orthodoxy* (1908)

> *I surround myself with obstacles.*
> *Whatever diminishes my discomfort diminishes my strength.*
> *The more constraints one imposes, the more one*
> *frees one's self of the chains that shackle the spirit.*
> — IGOR STRAVINSKY, *Poétique musicale sous forme de six leçons* (1939)

7.2.2.3. Constraint satisfaction. In Section 7.2.2.1 we solved numerous examples of XCC problems — exact covering with colors — which featured "items" and "options." Then in Section 7.2.2.2 we resolved lots of SAT problems — Boolean satisfiability — which featured "literals" and "clauses." All of these, and more, are instances of a combinatorial challenge that's more general yet, the *constraint satisfaction problem* — often called the CSP for short — which we will see is based on "variables," "domains," and "constraints."

The idea is simple: We're given a finite list of *variables* $(x_1, x_2, \ldots, x_n)$, to which we can assign values that belong to given finite *domains* $(D_1, D_2, \ldots, D_n)$. And we're also given a set of *constraints* $\{R_1, R_2, \ldots, R_m\}$, each of which specifies that a certain subset of the values $(x_1, x_2, \ldots, x_n)$ must be mutually compatible. Some combinations of values are "good"; the others are "nogood."

For example, let $n = 5$, and suppose that each domain is a set of letters:

$$D_1 = \{\mathtt{B},\mathtt{S}\},\;\; D_2 = \{\mathtt{C},\mathtt{L}\},\;\; D_3 = \{\mathtt{A},\mathtt{I},\mathtt{U}\},\;\; D_4 = \{\mathtt{E},\mathtt{O}\},\;\; D_5 = \{\mathtt{D},\mathtt{N}\}. \qquad (1)$$

Thus there are $2\times2\times3\times2\times2 = 48$ possible settings of $x_1x_2x_3x_4x_5$, from `BCAED` to `SLUON`. Let's also impose three constraints:

$$\begin{aligned} R_1(x_1,x_3,x_5) &= \text{`}x_1x_3x_5 \in \{\mathtt{BAN},\mathtt{BUD},\mathtt{SIN}\}\text{'};\\ R_2(x_1,x_4) &= \text{`}x_1x_4 \in \{\mathtt{BE},\mathtt{SE},\mathtt{SO}\}\text{'};\\ R_3(x_2,x_4,x_5) &= \text{`}x_2x_4x_5 \in \{\mathtt{COD},\mathtt{CON},\mathtt{LED}\}\text{'}. \end{aligned} \qquad (2)$$

This CSP has two solutions, easily found by hand (see exercise 1).

Every SAT problem is obviously a CSP in which all the domains are $\{0,1\}$. For example, problem $F = \{1\bar{2}, 23, \bar{1}\bar{3}, \bar{1}\bar{2}3\}$ in 7.2.2.2–(3) has four constraints,

$$\begin{aligned} &x_1x_2 \in \{00,10,11\};\quad x_2x_3 \in \{01,10,11\};\quad x_1x_3 \in \{00,01,10\};\\ &\qquad\qquad x_1x_2x_3 \in \{000,001,010,011,100,101,111\}. \end{aligned} \qquad (3)$$

Conversely, every CSP can be formalized as an equivalent SAT problem, by using several SAT variables to represent each CSP variable x whose domain size d exceeds 2. For example, if the domain is $\{0, 1, \ldots, d-1\}$, Section 7.2.2.2 discussed the "log encoding," with $l = \lceil \lg d \rceil$ Boolean variables meaning that $x = (x_{l-1} \ldots x_1 x_0)_2$. There's also the "direct encoding," with d variables $x_k = [x = k]$, as well as the "order encoding," which has $d-1$ variables $x^j = [x \geq j]$. We also discussed a variety of ways to represent arbitrary constraints, in the form of one or more clauses involving such Boolean variables. Each of those encodings has its own virtues and weaknesses, depending on the application.

Every XCC problem can, similarly, be regarded as a CSP. One way is to have a variable x_i for every primary item i, with domain D_i equal to the set of options that contain i. The constraints are that x_i and x_j cannot be options that conflict: If $x_i = o_i$ and $x_j = o_j$, where $o_i \neq o_j$, then o_i and o_j cannot have a common primary item, nor can they have a common secondary item that's colored differently in o_i and o_j. Conversely, exercise 7.2.2.1–100 presented one way to encode any CSP as an XCC problem.

Thus XCC, SAT, and CSP can each be reduced to the other two.

We've already learned how to construct excellent XCC solvers and excellent SAT solvers, so we might be tempted to stop there, regarding CSP as a problem that's already been well solved. But we shall see that careful consideration of the CSP not only clarifies XCC and SAT, it also teaches us important new methods.

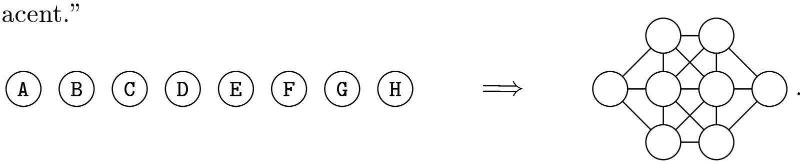
Related models. Many groups of researchers have independently adopted conceptual frameworks that are identical to or very similar to the notions of variables, domains, and constraints. For example, a theory of *relational structures* has been developed as part of the branch of mathematics called "model theory," which spawned "universal algebra." A relational structure is a set D together with a set $\{R_1, R_2, \ldots\}$ of relations or "predicates" defined on the elements of D. Each relation R_i depends on k elements, for some $k = k_i$, and it defines the k-tuples of elements for which that predicate is true. [See P. M. Cohn, *Universal Algebra* (1965), Chapter V.]

Let's be a little more precise. The *Cartesian product* of sets $(D_1, \ldots, D_n)$, denoted by $D_1 \times \cdots \times D_n$, is the set of all n-tuples $(x_1, \ldots, x_n)$ such that $x_i \in D_i$ for $1 \leq i \leq n$. Thus, $D_1 \times \cdots \times D_n$ is the set of all solutions to a CSP with domains $(D_1, \ldots, D_n)$, in the case when there are no constraints. An n-tuple such as $(x_1, \ldots, x_n)$ is often written simply as $x_1 \ldots x_n$, when commas aren't necessary. We also write $D \times \cdots \times D = D^n$ when the n domains are all identical.

A k-ary *relation* on sets $(D_1, \ldots, D_k)$ is a subset of $D_1 \times \cdots \times D_k$. We write either $R(x_1, \ldots, x_k)$ or $x_1 \ldots x_k \in R$ when we want to say that the k-tuple $(x_1, \ldots, x_k)$ satisfies relation R. The relation is called *binary* when $k = 2$, *ternary* when $k = 3$, *quaternary* when $k = 4$, and so on; it's *unary* when $k = 1$. (Strictly speaking, there also are *nullary* relations; see exercise 5.)

The simplest nontrivial relational structures arise where there's just a single binary relation. In fact, this case is so simple, we hardly ever think of it as a "relation structure" at all: We call it a *directed graph.* Indeed, we know well

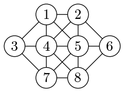
A simple example. To warm up, let's look at a little puzzle that appeared on a British TV show called *The Crystal Maze* in 1994. The task is simple—but you've got only two minutes to do it: "Place eight large disks, marked with the letters `A` through `H`, onto the eight circles shown; consecutive letters can't be adjacent."



(11)

We're actually facing two challenges here, namely (i) solve the puzzle; and (ii) express it as a constraint satisfaction problem, so that a computer can solve it for us. We'll tackle (ii), so as not to spoil the fun of (i). And we'll allow ourselves ten minutes, say, to accomplish goal (ii).

What are appropriate variables, domains, and constraints? We'd better label the vertices of the graph, so that we can readily describe what we want to define. One approach, based on the labeling shown, is to have eight variables $\{x_1, x_2, \ldots, x_8\}$, one for each vertex, each with domain $\{\texttt{A}, \texttt{B}, \ldots, \texttt{H}\}$. Then there are seventeen constraints, one for each edge of the graph; for example, the constraint for edge $1 — 2$ is



$$x_1x_2 \in \{\texttt{AC},\texttt{AD},\texttt{AE},\texttt{AF},\texttt{AG},\texttt{AH},\texttt{BD},\texttt{BE},\texttt{BF},\texttt{BG},\texttt{BH},\texttt{CA},\texttt{CE},\texttt{CF},\texttt{CG},\texttt{CH},\texttt{DA},\texttt{DB},\texttt{DF},\texttt{DG},\texttt{DH},\texttt{EA},\\ \texttt{EB},\texttt{EC},\texttt{EG},\texttt{EH},\texttt{FA},\texttt{FB},\texttt{FC},\texttt{FD},\texttt{FH},\texttt{GA},\texttt{GB},\texttt{GC},\texttt{GD},\texttt{GE},\texttt{HA},\texttt{HB},\texttt{HC},\texttt{HD},\texttt{HE},\texttt{HF}\}, \tag{12}$$

and the same relation is used for all of the other edges. It can be written much more succinctly, if we assume that the letters are represented by integer codes:

$$|x_1 - x_2| \;>\; 1. \tag{13}$$

OK, that took three minutes. Are we done? Well, no, actually; the seventeen constraints we've specified do not obviously rule out the possibility that $x_1 = x_8$. We're not allowed to put a disk on two different circles.

We could add eleven further constraints, namely $x_i \neq x_j$ for each of the yet-unconstrained pairs. But seasoned CSP solvers generally prefer to append a single *global* constraint instead, involving all of the variables at once:

$$x_1,\ x_2,\ x_3,\ x_4,\ x_5,\ x_6,\ x_7,\ x_8 \text{ are all different.} \tag{14}$$

Indeed, special methods have been devised for the "all-different" constraint, because it arises in so many different problems. With (14), we've satisfied (ii).

Five minutes to go. Is there a better way? Another possibility is to let the variables be $\{\texttt{A}, \texttt{B}, \ldots, \texttt{H}\}$, one for each *disk*, each with domain $\{1, 2, \ldots, 8\}$. Then only *seven* constraints are needed, one for each pair of consecutive letters; e.g.,

$$\texttt{AB} \in \{16,17,18,23,27,28,32,35,36,38,46,53,61,63,64,67,71,72,76,81,82,83\}. \tag{15}$$

And each of these constraints has only 22 tuples, compared to 42 in (12). It's a win! Of course we also need the global all-different constraint. (See exercise 13.)

If we only had more time, we could have discovered a completely different way to model problem (11) as a CSP, such as the approach in exercise 16.

Automating automobiles. We've already seen dozens and dozens of significant examples of constraint-based problems when we studied exact covering and SAT. But we certainly haven't exhausted all of the major applications, and several problems on our yet-unexamined list have been associated historically with the CSP. One of them, known as the *car sequencing problem*, is especially appropriate for us to study next, not only because its initials are "CSP" but also because it is problem 001 in CSPLIB — a noteworthy collection of benchmarks that was launched by I. P. Gent and T. Walsh in 1999 (see *LNCS* **1713** (1999), 480–481).

Consider the portion of an automobile assembly line where optional features are being installed on newly made vehicles. Some of the cars will be made with moonroofs; some will have heated seats; and so on. The assembly line is divided into work areas, one for each special feature. Work area w has space for q_w cars, where q_w is the number of time slots needed to install feature w as the conveyor belt moves the cars along. If at most p_w/q_w of the cars need that feature, p_w installers are on duty, one of whom will commence work when a car enters the area and walk with it until the installation is done. The car sequencing problem is the task of arranging a given set of cars into a sequence so that *no subsequence of* q_w *consecutive cars will include more than* p_w *that need feature* w.

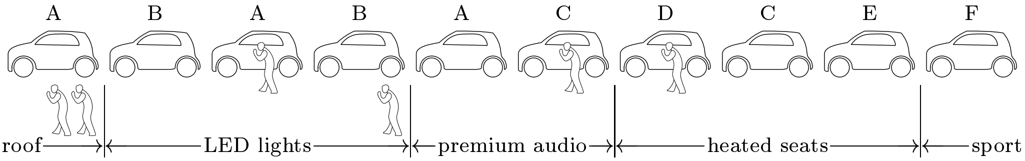


Fig. 100. Cars of models A, B, ... enter this assembly line at the far right, receiving optional features when they're in an appropriate work area. If this sequence has specifications (16), the final car (F) will be delayed in the LED area, because three cars in a row want that feature. The car sequencing problem tries to avoid such delays.

For example, there might be six models using the following subsets of five features:

Model	A	B	C	D	E	F	w	p_w	q_w
moonroof?	☐	☑	☐	☑	☑	☐	0	2	5
LED lights?	☑	☐	☑	☐	☑	☑	1	2	3
premium audio?	☐	☑	☑	☐	☐	☑	2	1	2
heated seats?	☐	☐	☐	☑	☐	☑	3	1	3
sport suspension?	☐	☐	☐	☐	☑	☐	4	1	5

(16)

Suppose ten cars of models $\{A, A, A, B, B, C, C, D, E, F\}$ are to be made. The sequence ABABACDCEF is almost correct, but it fails on the final car (see Fig. 100). Can you find a delay-free sequence? Notice that the left-right reflection of any solution is also a solution; we can rule out mirror images by requiring that model F, say, appears among the first five cars. Exercise 17 has the (unique) answer.

The car sequencing problem has boundary effects at the left and right that make it somewhat unrealistic. (Industrial assembly lines don't really start out empty every day!) Still, it's a nice clean problem, instructive to chew on.

An international competition was held in 2005, based on actual industrial data. It included additional constraints, such as the colors of paint to be used and the initial contents of the assembly line, and it inspired many creative solutions. [See C. Solnon, V. D. Cung, A. Nguyen, and C. Artigues, *EJOR* **191** (2008), 912–927.] The winning programs were based on local search methods analogous to WalkSAT, using “greedy” heuristics.

Line labeling in computer vision. Speaking of history, let’s turn now to some fascinating aspects of computer vision that influenced much of the early work on constraint processing. When a camera photographs a scene, it makes a two-dimensional image of three-dimensional reality; interesting problems arise when we try to reconstruct the reality from the image.

We’ll work with an extremely simplified yet powerful model, as the original researchers did: Our “reality” will be a world of special polyhedral objects, where exactly *three faces* meet at each of the vertices. For example, an ordinary cube or tetrahedron or  will qualify. But an octahedron will not, nor will an Egyptian-style pyramid, nor , because a vertex where four faces meet isn’t allowed. These three-faced concepts can be generalized, of course, but it’s helpful to start with a thorough understanding of the comparatively simple trihedral world.

More precisely, the 3D objects we shall deal with have no curved surfaces. They are defined by vertices, edges, and faces, where the vertices are “corners” at which edges and faces come together. All of the faces are “flat,” meaning that their points all lie on some plane. Each face is bounded by an exterior polygon, possibly with one or more interior polygons delimiting “holes” in the face. Each edge runs between two vertices and is part of the (infinite) line where the planes of two adjacent faces meet; it’s a segment of the polygonal boundaries of those faces. And significantly, *each vertex is the endpoint of exactly three edges.* We shall call such an object a *three-valent polyhedral object*, or 3VP for short. (See Fig. 101.)

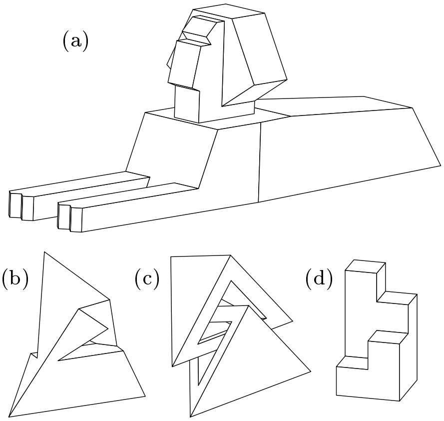


Fig. 101. Examples of 3VPs (three-valent polyhedra): (a) A stylized sphinx. [68 vertices, 102 edges, 38 faces.] (b) The Szilassi polyhedron, defined in exercise 26. Each of its seven faces is adjacent to all of the other six(!). [14 vertices, 21 edges, 7 faces.] (c) A clasp formed from two identical, interlocked objects, each of which is a tetrahedron from which a large triangular wedge has been hollowed out. [20 vertices, 30 edges, 14 faces.] (d) The histoscape for the matrix $\left(\begin{smallmatrix}4&3\\1&2\end{smallmatrix}\right)$, as defined in exercise 27. [20 vertices, 30 edges, 12 faces.] Many of the vertices, edges, and faces of these examples are invisible because they lie behind the parts that we *can* see.

The two-dimensional images shown here make sense to us, somehow, although significant depth information has been lost. In some mysterious way we’ve learned to rely on visual cues in order to understand what’s really present.

INDEX AND GLOSSARY

> *Index making has been held to be the driest*
> *as well as lowest species of writing.*
> *We shall not dispute the humbleness of it;*
> *but since we have had to make an Index ourselves,*
> *we have discovered that the task need not be so very dry.*
>
> — LEIGH HUNT, in *The Indicator* (1819)

When an index entry refers to a page containing a relevant exercise, see also the *answer* to that exercise for further information. An answer page is not indexed here unless it refers to a topic not included in the statement of the exercise.

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