chapter 8

Technology and School Mathematics

Technology is an essential tool for teaching and learning mathematics effectively; it extends the mathematics that can be taught and enhances students' learning.

www.nctm.org/about/position_statements/position_statement_13.htm

The term *technology* in the context of school mathematics refers primarily to calculators of all sorts and computers, including access to the Internet and the available resources for use with these devices. The simple position statement of the NCTM (just quoted) is quite clear with regard to technology: It is an *essential tool* for both learning and teaching mathematics. It is important not to think of technology as an extra burden added on to the list of things you are trying to accomplish in your classroom. Rather, technology should be another of the many real tools at your disposal for helping children learn mathematics. Seen as an integral part of your instructional arsenal of tools for learning, technology can enlarge the scope of the content students can learn and broaden the range of problems that students are able to tackle (Ball & Stacey, 2005; NCTM Position Statement, 2003).

That technology is one of the six principles in the *Principles and Standards* document further highlights the importance that NCTM gives to technology (see Chapter 1).

Calculators and computer software (including Internet-based applications, or "applets") are highlighted throughout this text, with references to specific activities and programs where they are appropriate. The purpose of this chapter is to examine technology and the teaching of mathematics in a more general way so that you will be able to make informed judgments about truly integrating technology into your array of instructional tools.

Calculators in the Mathematics Classroom

Mathematics educators have long understood the value of calculators in the study of mathematics. Since 1976, NCTM has published numerous articles, books, and position statements, all advocating the regular use of calculators in the teaching of mathematics at all grade levels. In its 2005 position statement on computation and calculators, NCTM clarified its long-standing view that there is an important place in the curriculum for both calculator use and the development of a variety of computational skills (www.nctm.org).

Unfortunately, the everyday use of calculators in society, coupled with professional support of calculators in schools, has had a muted impact on the mathematics classroom, especially at the elementary level. Resistance to the use of calculators has diminished but not disappeared. The vocal minority of detractors to the reform movement often assail the use of calculators as "dumbing down" the curriculum or as a "crutch." Their inflammatory rhetoric often resonates with parents, who want what is best for their children. Parents must be made aware of the fact that calculator use will in no way prevent children from learning mathematics; in fact, calculators used thoughtfully and appropriately can enhance the learning of mathematics. Furthermore, parents must learn that the use of both calculators and computers requires the student to be a "problem solver." Calculators always calculate according to the input information. Calculators cannot substitute for understanding.

Benefits of Calculator Use

Rather than fearing potential damage that calculators might do, it is important to understand how calculators

can contribute to the learning of mathematics. In this section, the focus is on simple calculators. A discussion of graphing calculators is reserved for later.

Calculators Can Be Used to Develop Concepts

The calculator can be much more than a device for calculation. It can be used effectively to develop concepts. *Adding It Up: Helping Children Learn Mathematics* (NRC, 2001) cites several long-term studies that have shown that students in grades 4–6 who used calculators improved their conceptual understanding. Activities for developing concepts with the calculator are suggested throughout this book, especially in the areas of numeration and computation. Here are two simple examples. On the calculator, 796 ÷ 42 = 18.95348. Consider the task of using the calculator to determine the whole-number remainder. Another example is to use the calculator to find a number that when multiplied by itself will produce 43. In this situation, a student can press 6.1 [×] [=] to get the square of 6.1. For students who are just beginning to understand decimals, the activity will demonstrate that numbers such as 6.3 and 6.4 are between 6 and 7. Furthermore, 6.55 is between 6.5 and 6.6. For students who already understand decimals, the same activity serves as a meaningful and conceptual introduction to square roots.

Calculators Can Be Used for Drill

The calculator is an excellent drill device that requires no computer or software. For example, students who want to practice the multiples of 7 can press 7 [×] 3 and delay pressing the [=]. The challenge is to answer the fact to themselves before pressing the [=] key. Subsequent multiples of 7 can be checked by simply pressing the second factor and the [=]. The TI-10 and TI-15 calculators now have built-in problem-solving modes in which students can practice facts, develop lists of related facts, and test equations or inequalities with arithmetic expressions on both sides of the relationship symbol (http://education.ti.com/us/product/tech/10/features/1015getstart.html).

A class can be split in half with one half required to use a calculator and the other required to do the computations mentally. For 3000 + 1765, the mental group wins every time. It will also win for simple facts and numerous problems that lend themselves to mental computation. Of course, there are many computations, such as 537 × 32, where the calculator is preferred and that side will win. Not only does this simple exercise provide practice with mental math, but it also demonstrates to students that it is not always appropriate to reach for the calculator. Studies have found that for average-ability learners, calculator use enhanced basic skills acquisition (NRC, 2001).

Calculators Enhance Problem Solving

Several research studies have found that calculator use improved the problem-solving abilities of learners at all ability levels for all grades (NRC, 2001). The mechanics of computation can often distract students' attention from the meaning of the problem they are working on. As students come to understand the meanings of the operations, they should be exposed to realistic problems with realistic numbers. The numbers may be beyond their abilities to compute, but the calculator makes these realistic problems accessible.

Calculators Save Time

By-hand computation is time consuming, especially for young students who have not developed a high degree of mastery. Why should time be wasted having students add numbers to find the perimeter of a polygon? Why compute averages, find percents, convert fractions to decimals, or solve problems of any sort with pencil-and-paper methods when computation skills are not the objective of the lesson?

Calculators Are Commonly Used in Society

Nowadays, almost everyone uses calculators in every facet of life that involves any sort of exact computation—everyone except schoolchildren. Students should be taught how to use this commonplace tool effectively and also learn to judge when it is appropriate to use it. Many adults have not learned how to use the automatic constant feature of a calculator and are not practiced in recognizing gross errors that are often made on calculators. Effective use of calculators is an important skill that is best learned by using them regularly and meaningfully.

Addressing Myths and Fears About Using Calculators

The lingering opposition to calculators is largely based on misinformation. Myths and fears about students not learning because of using calculators still persist, even in the face of evidence to the contrary.

Myth: If Kids Use Calculators, They Won't Learn the "Basics"

Every advocate of calculator use must make it clear to parents that basic fact mastery and flexible computational skills, including mental computation, remain important goals of the curriculum. By and large, research has demonstrated that the availability of calculators has no negative effect on traditional skills (NRC, 2001). Although the eighth NAEP assessment data suggest a decrease in achievement for fourth graders who use calculators either weekly or every day, it is important to note that the same data also show that only 5 percent of teachers report everyday calculator use and only 21 percent report weekly use (http://nces.ed.gov/nationsreportcard/mathematics/results). Moreover, evidence from a meta-analysis of calculator use shows a slight negative effect of calculators among fourth graders but not among students of any other grade (NRC,

2001). This may be an artifact of conditions specific to those studies of calculator use that included fourth graders. Most important, the performance of tedious by-hand computations does not involve thinking or reasoning or solving problems. Employers want employees who can think and solve novel problems.

Myth: Calculators Make Students Lazy

Almost no mathematical thinking is involved in doing routine computations by hand. People who use calculators when solving problems are, therefore, using their intellect in more important ways—reasoning, conjecturing, testing ideas, and solving problems. When used appropriately, calculators enhance learning; they do not get in the way of learning.

Myth: Students Should Learn the "Real Way" Before Using Calculators

Following rules for pencil-and-paper computation does little to help students understand the ideas behind them. A glaring example is the invert-and-multiply method for division of fractions. Few parents and elementary teachers can explain why this method makes sense. And yet they all had extensive practice with that technique. To one degree or another, the same is true of nearly all computational procedures.

It is essential to point out that by-hand techniques are not to be totally abandoned and that introductory explorations are often best done without a calculator. The teacher must play a role in setting the necessary explorations in the classroom.

Myth: Students Will Become Overly Dependent on Calculators

Calculators kept from students are like forbidden fruit. When finally allowed to use them, students often use them for the simplest of tasks. Teachers in the upper grades often complain that students are using their calculators all the time.

It is essential that mastery of basic facts, mental computation, and some attention to by-hand techniques continue to be requirements for all students. In lessons in which these skills are the objective, the calculator should simply be off limits. When students learn these essential noncalculator skills, they rarely use the calculator inappropriately. Furthermore, if the calculator is always available for appropriate uses, students learn when and how to use it wisely.

Calculators for Every Student, Every Day

Calculators should be in or on students' desks at all times from kindergarten through high school.

In addition to the benefits already described, here are a few arguments in favor of calculator access at all times:

- First and foremost, it does no harm. Any teacher can conduct an activity or pose tasks in which calculators are set off limits. Availability of calculators does not detract from the development of basic skills.
- Many excellent explorations that happen spontaneously in a problem-solving environment will be enhanced by the use of calculators. Students should not have to leave their desks or ask permission to use a calculator when solving a problem.
- When calculators are kept from students, they tend to be used for special "calculator lessons," promoting the student belief that calculators are not common tools for solving problems.
- Students must learn to make wise choices about when to use calculators—for tedious computations—and when to use mental math—for simple computations and estimations. They learn this only by making such choices independently and on a regular basis.

Graphing Calculators

The graphing calculator, once thought useful only in high school, is so important to middle school mathematics that it deserves some special attention. Today, a graphing calculator makes sense for all middle school students. Cost is still a possible deterrent. Several models are available for under $75, less than the cost of a pair of sneakers or a few CDs. A calculator purchased in the sixth grade may be the only one the student will need through high school. A school can purchase a classroom set for less than the price of a single computer.

What the Graphing Calculator Offers

It is a mistake to think that graphing calculators are only for doing "high-powered" mathematics usually studied by honors students in high school. Here is a list of some features the graphing calculator offers, every one of which is useful within the standard middle school curriculum.

- The display window permits compound expressions such as $3 + 4(5 - 6/7)$ to be shown completely before being evaluated. Furthermore, once evaluated, previous expressions can be recalled and modified. This promotes an understanding of notation and order of operations. The device is also a significant tool for exploring patterns and solving problems. Expressions can include exponents, absolute values, and negation signs, with no restrictions on the values used. (Note that this same feature is now found on many simpler calculators such as the TI-15.)
- Even without using function definition capability, students can insert values into expressions or formulas

without having to enter the entire formula for each new value. The results can be entered into a list or table of values and stored directly on the calculator for further analysis.

- Variables can be used in expressions and then assigned different values to see the effect on expressions. This simple method helps with the idea of a variable as something that varies.
- The distinction between "negative" and "minus" is clear and very useful. A separate key is used to enter the negative of a quantity. The display shows the negative sign as a superscript. If $^{-}5$ is stored in the variable B, then the expression $^{-}2 - {}^{-}B$ will be evaluated correctly as $^{-}7$. This feature is a significant aid in the study of integers and variables.
- Points can be plotted on a coordinate screen either by entering coordinates and seeing the result or by moving the cursor to a particular coordinate on the screen.
- Very large and very small numbers are managed without error. The calculator will quickly compute factorials, even for large numbers, as well as permutations and combinations. Graphing calculators use scientific notation so that large and small numbers do not result in error statements. For example, $23! = 1.033314797 \times 10^{40}$.
- Built-in statistical functions allow students to examine the means, medians, and standard deviations of large sets of realistic data without a computer. Data are easily entered, ordered, added to, or changed almost as easily as on a spreadsheet.
- Graphs for data analysis are available, including box-and-whisker plots, histograms, and, on some calculators, pie charts, bar graphs, and pictographs.
- Random number generators allow for the simulation of a variety of probability experiments that would be difficult without such a device.
- Scatter plots for ordered pairs of real data can be entered, plotted, and examined for trends. The calculator will calculate the equations of best-fit linear, quadratic, cubic, or logarithmic functions.
- Functions can be explored in three modes: equation, table, and graph. Because the calculator easily switches from one to the other and because of the trace feature, the connections between these modes become quite clear. Even sixth-grade students can explore a variety of types of functions along with their graphs and function tables. There is no need to wait until high school to let students explore how the m and b in $y = mx + b$ affect the graph.
- The graphing calculator is programmable. Programs are very easily written and understood. For example, a program involving the Pythagorean theorem can be used to find the length of sides of right triangles.
- Data, programs, and functions can be shared from one graphing calculator to another and, thus, to the display calculator for the overhead or TV monitor. This permits students to share and discuss work with the rest of the class. Calculators also connect to computers to store data and programs and to print out anything that can be seen on the calculator screen.
- Students can share data from one calculator to another, connect their calculators to a classroom display screen, save information on a computer, and download software applications that give additional functionality for special uses.

Most of the ideas on this list are explored briefly in appropriate chapters in this book.

Arguments against graphing calculators are similar to those for other calculators—and are equally hollow and unsubstantiated. These amazing tools have the potential of significantly opening up real mathematics for students. It is time that graphing calculators became a regular tool in the middle grades.

Electronic Data Collection

In addition to the capabilities of the graphing calculator alone, electronic data collection devices make them even more remarkable. Texas Instruments calls its version the CBL (for *computer-based laboratory*), and such calculators are often referred to by that acronym. The current Texas Instruments version is the CBL-2. Casio's current version is called the EA-200 and is nearly identical in design. These devices accept a variety of data collection probes, such as temperature or light sensors and motion detectors, that can be used to gather real physical data. The data can be transferred to the graphing calculator, where they are stored in one or more lists. The calculator can then produce scatter plots or prepare other analyses. With appropriate software, the data can also be transferred to a computer.

A number of excellent resource books are available that describe experiments in detail. Most include disks with calculator programs that make the interface with the CBL quite easy. With a CBL, science and mathematics meet head-on.

The most popular probe for mathematics teachers is the motion detector. Texas Instruments has a special motion detector called a Ranger or CBR that connects directly to the calculator without requiring a CBL unit. Experiments with a motion detector include analysis of objects rolling down an incline, bouncing balls, or swinging pendulums. The motion actually determines the distance an object is from the sensor. When distance is plotted against time, the graph shows velocity. Students can plot their own motion walking toward or away from the detector. The concept of rate when interpreted as the slope of a distance-to-time curve can become quite dramatic.

Though not as widely used as calculators, personal digital assistants (PDAs) such as the PalmPilot or Handspring

Visor have similar capabilities to the CBL. As with the CBL, PDAs can use software and available probes to collect and analyze data. The ImagiProbe from ImagiWorks (www.imagiworks.com) connects to a PDA for distance and temperature data. The Technology Enhanced Elementary and Middle School Science site from the Concord Consortium (www.concord.org/teemss) offers strategies and ideas for using these devices.

The Computer as a Tool in Mathematics

Tool software is a generic term for software that performs a function that makes doing something easier. A very common software tool is the word processor. Other popular tools include spreadsheets, databases, and presentation software such as PowerPoint. A number of powerful tools have been created for use in the mathematics classroom. These exist in two formats: as stand-alone programs that can be purchased from software publishers and as Internet-based applications or *applets* (for "little applications") accessible through Web browsers such as Netscape Browser, Microsoft Internet Explorer, and Apple's Safari.

Applets are always much smaller, more targeted programs than commercial software. A significant advantage is that they are freely accessed on the Internet. Many can also be downloaded so that an Internet connection is not required for student use. Some of these applets are described briefly throughout this book and at the end of each chapter. At the end of this chapter, sites are listed that collectively offer well over 100 applets. You are strongly urged to browse and play. Many of these are lots of fun!

In addition to applets on the Web, another excellent source is the CD-ROMs that come with every volume of the NCTM *Navigations* series. This newest collection of resources from NCTM is designed to help teachers translate *Principles and Standards for School Mathematics* into the classroom. As of 2005, grade-band books are available on the topics of Algebra, Geometry, Number and Operations, Data Analysis, Probability, and Problem Solving and Reasoning. Most of the applets on the *Navigations* CDs can also be found on NCTM's *Illuminations* Web site.

A mathematical software tool is somewhat like a physical manipulative; by itself, it does not teach. However, the user of a well-designed tool software package has an electronic "thinker toy" with which to explore mathematical ideas.

Electronic Manipulatives for Numeration

In these programs, screen versions of popular manipulative models for counting, place value, and fractions are available for students to work with freely without the computer posing problems, evaluating results, or telling the students what to do.

At the earliest level, there are programs that provide "counters" such as colored tiles, pictures of assorted objects, or in one specific case, Unifix cubes.* Typically, students can drag counters to any place on the screen, change the colors, and put them in groupings. Some programs have options that turn on counters for the screen or subsets of the screen. Nonmathematical programs such as *Kid Pix Deluxe 4 for Schools* (Riverdeep, 2005) can also be used to "stamp" discrete objects on the screen, explore shapes, word process, and more.

Base-ten blocks (ones, tens, and hundreds models), assorted fraction pieces, and Cuisenaire rods (centimeter rods) are available in some software packages as well as in Web-based applets. These include both pure tool programs and instructional software programs that attempt to teach or tutor. Some fraction models are more flexible than physical models. For example, a circular region might be subdivided into many more fractional parts than is reasonable with physical models. When the models are connected with on-screen counters, it is possible with some programs to have fraction or decimal representations shown so that connections between fractions and decimals can be illustrated. *Mighty Math Number Heroes* (Riverdeep, 2005) does a nice job of connecting these types of representations for fractions with denominators as small as twelfths. Many of these commercial programs, however, are not completely open to student use without some constraints. For example, in *MathKeys: Unlocking Whole Numbers, Grades 3–5* (Riverdeep, 2005), pieces can be combined or taken apart only by adding or subtracting two quantities.

Web-based tools or applets exist that are designed so that students may manipulate them without constraint. For example, the Base Ten Block Applet (www.arcytech.org/java/b10blocks/b10blocks.html) allows children to collect as many flats, rods, and units as they wish, gluing together groups of ten, or breaking a flat into ten rods or a rod into ten units. Other than providing a description of the pieces, there is no attempt to teach or tutor.

The obvious question is, Why not simply use the actual physical models? Especially for electronic tools (as opposed to instructional software), electronic or virtual manipulatives have some advantages that merit integrating them into your instruction—not just adding them on as extras.

- *Qualitative Differences in Use.* Usually it is at least as easy to manipulate virtual manipulatives as it is to use their physical counterparts. However, control of

*Unifix® is the registered trademark for a set of plastic connecting cubes. The cubes are about 2 cm in width and snap easily into bars of any length. They are popular materials for grades K–2.

materials on the screen requires a different, perhaps more deliberative, mental action that is "more in line with the *mental actions* that we want children to carry out" (Clements & Sarama, 2005, p. 53). For example, the base-ten rod representing a ten can be broken into 10 single blocks by clicking on it with a hammer icon. With physical blocks, the ten must be traded for the equivalent blocks counted out by the student.

- *Connection to Symbolism.* Most virtual manipulatives for number include dynamic numerals or odometers that change as the representation on the screen changes. This direct and immediate connection to numeral representation is impossible with physical models.
- *Unlimited Materials with Easy Cleanup.* With virtual manipulatives, a student can easily erase the screen and begin a new problem with the click of a mouse. He or she will never run out of materials. For place value, even the large 1000 cubes are readily available in quantity. And there is no storage or cleanup to worry about.
- *Accommodations for Special Purposes.* For English language learners, some programs come with speech enhancements so that the students hear the names of the materials or the numbers. Some programs and applets are available in Spanish. For students with physical disabilities, the computer models are often easier to access and use than physical models.

Many software-based programs also offer a word-processing capability connected to the workspace. This allows students to write a sentence or two to explain what they have done or perhaps to create a story problem to go with their work. Printing a picture of the workspace, with or without a written attachment, creates a record of the work for the teacher or parent that is not possible with physical models. Web-based applets typically do not have print capabilities.

Geometry Tools

Computer tools for geometric exploration are much closer to pure tools than those just described for numeration. That is, students can use most of these tools without any constraints. They typically offer some significant advantages over physical models, although the computerized tools should never replace physical models in the classroom.

Blocks and Tiles

Programs that allow students to "stamp" geometric tiles or blocks on the screen are quite common. Typically, there is a palette of blocks, often the same as pattern blocks or tangrams, from which students can choose by clicking the mouse. Often the blocks can be made "magnetic" so that when they are released close to another block, the two will snap together, matching like sides. Blocks can usually be rotated, either freely or in set increments. Different programs offer different variations and features. Figure 8.1 shows a simple yet powerful applet that permits a student to slice any of the three shapes in any place and then manipulate any of the pieces. This is a good example of something a student can do with a computer that would be difficult or impossible with physical models. You may find the following:

- The ability to enlarge or reduce the size of blocks, usually by set increments
- The ability to "glue" blocks together to make new blocks
- The ability to reflect one or more blocks across a mirror line or to rotate them about a point
- Puzzle tasks built into the program
- The ability to measure area or perimeters
- The ability to select polygons with a variable number of sides
- The possibility of creating three-dimensional shapes and rotating them in space

For students who have poor motor coordination or a physical disability that makes block manipulation difficult, the computer versions of blocks are a real plus. Colorful printouts can be displayed, discussed, and taken home if that option is available.

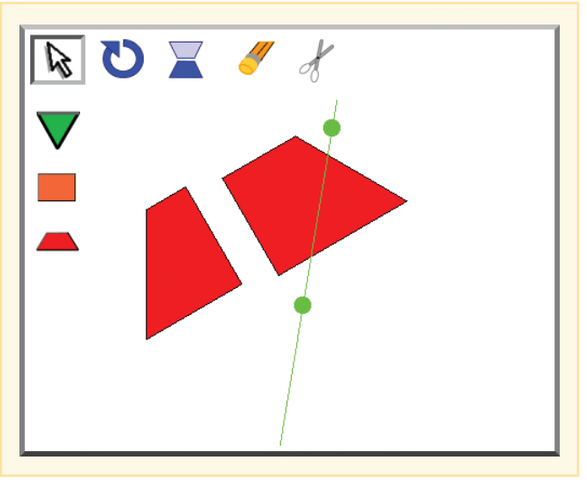

FIGURE 8.1 The "Cutting Shapes Tool."

Used with permission from the CD-ROM included with the NCTM pre-K–2 *Navigations* book for geometry by C. R. Findell, M. Small, M. Cavanagh, L. Dacey, C. E. Greenes, and L. J. Sheffield. Copyright © 2001 by the National Council of Teachers of Mathematics, Inc. All rights reserved. The presence of the screenshot from *Navigations* does not constitute or imply an endorsement by NCTM.

Drawing Programs

For younger students, drawing shapes on a grid is much easier and more useful for geometric exploration than free-form drawing. Several programs offer electronic geoboards on which lines can be drawn between points on a grid. When a shape such as a triangle is formed, it can typically be altered just as you would a rubber band on a geoboard. For examples, check the NCTM e-*Standards* or NCTM's Illuminations Web site. Addresses are at the end of this chapter. The electronic geoboard programs offer a larger grid on which to draw, ease of use, and the ability to save and print. Some include measuring capabilities as well as reflection and rotation of shapes, things that are difficult or impossible to do on a physical geoboard. An example of a good Internet applet for drawing is the Isometric Drawing Tool found at NCTM's Web site (see Figure 8.2).

Dynamic Geometry Software

Dynamic geometry programs are much more than simple drawing packages. These exciting programs allow students to create shapes on the computer screen and then manipulate and measure them by dragging vertices. The most well-known programs of this type are *The Geometer's Sketchpad* (Key Curriculum Press) and *Cabri Geometry II* (Texas Instruments). Dynamic geometry programs allow the creation of geometric objects (lines, circles) so that their relationship to another screen object is established. For example, a new line can be drawn through a point and perpendicular to another line. A midpoint can be established on any line segment. Once created, these relationships are preserved no matter how the objects are altered. Once thought of as programs for high school students, they are now commonly used in middle school classrooms and are appropriate in classrooms even as early as grade 3. Dynamic geometry software can dramatically both change and improve the teaching of geometry. The ability of students to explore geometric relationships with this software is unmatched with any noncomputer mode. More detailed discussion of these programs can be found in Chapter 21.

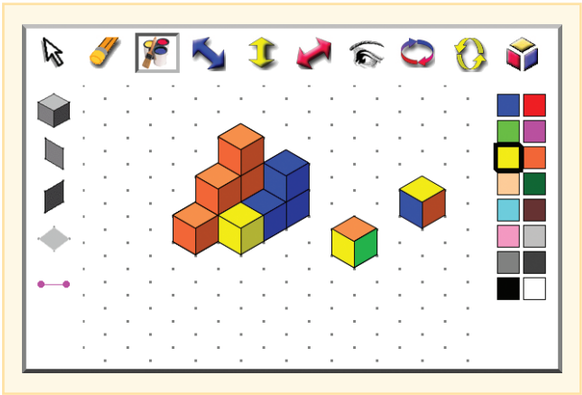

FIGURE 8.2 The "Isometric Drawing Tool" applet from NCTM's Illuminations Web site.

Probability and Data Analysis Tools

These computer tools allow for the entry of data and a wide choice of graphs made from the data. In addition, most will produce typical statistics such as mean, median, and range. Some programs are designed for students in the primary grades. Others are more sophisticated and can be used through the middle grades. These programs make it possible to change the emphasis in data analysis from "how to construct graphs" to "which graph best tells the story."

It should be noted that the spreadsheet and the graphing calculator provide much the same capabilities as dedicated data graphing software. Generally, the data programs, described in Chapter 22, offer more graphing options and easier use than a spreadsheet since they are designed to assist in the development of data analysis concepts.

Probability Tools

These programs, also described in Chapter 23, make it easy to conduct controlled probability experiments and see graphical representations of the results. The young student using these programs must accept that when the computer "flips a coin" or "spins a spinner," the results are just as random and have the same probabilities as if done with real coins or spinners. The value of these programs is found in the ease with which experiments can be designed and large numbers of trials conducted.

Spreadsheets and Data Graphers

Spreadsheets are programs that can manipulate rows and columns of numeric data. Values taken from one position in the spreadsheet can be used in formulas to determine entries elsewhere in the spreadsheet. When an entry is changed, the spreadsheet updates all values immediately.

Because the spreadsheet is among the most popular pieces of standard tool software outside of schools, it is often available in integrated packages you may already have on your computer. The spreadsheet program *Excel* is available separately or included in the Microsoft *Office* suite. A spreadsheet similar to *Excel* is found in the *Apple Works* programs. Students as early as third grade can use these programs to organize data, display data graphically in

various ways, and do numeric calculations such as finding the total or the mean. Students only need to know how to use the capabilities of the spreadsheet that they will be using. These functions are well within the grasp of the elementary student.

As an alternative to these commercial packages, the *Illuminations* Web site from NCTM offers a couple of very nice spreadsheet Internet applets, *Spreadsheet* and *Spreadsheet and Graphing Tool*. They can be used while connected to the Internet, or they can be downloaded to your computer.

Programs that do the tedious work of creating graphs will permit students in the primary grades to focus on how different graphs convey the information they have gathered. Nice examples in this category are *The Graph Club* for students in grades K to 3 and *Graph Master* for grades 4 to 8 (both from Tom Snyder Productions). *TinkerPlots* (Key Curriculum Press) is a new program developed with NSF funding for students in grades 4 to 8. This program not only creates graphs but also encourages the manipulation of data so that students learn statistical concepts and big ideas about the shape of data.

A number of very nice applets that offer graphing capabilities are freely available on the Web.

Function Graphers

Function graphing software permits the user to create the graph of almost any function very quickly. Multiple functions can be plotted on the same axis. It is usually possible to trace along the path of a curve and view the coordinates at any point. The dimensions of the viewing area can be changed easily so that it is just as easy to look at a graph for x and y between -10 and $+10$ as it is to look at a portion of the graph thousands of units away from the origin. By "zooming in" on the intersection of two graphs, it is possible to find points of intersection without algebraic manipulation. Similarly, the point where a graph crosses the axis can be found to as many decimal places as is desired.

All of the features just described are available on all graphing calculators. Computer programs add speed, color, visual clarity, and a variety of other interesting features to help students analyze functions.

Instructional Software

Instructional software is designed for student interaction in a manner similar to the textbook or a tutor. It is designed to teach. The distinction between tool and instructional software is not always clear since some packages include a tool-only component. Nor is it always clear how to categorize particular instructional programs. In the following discussion, the intent is to provide some perspective on the different kinds of input to your mathematics program that instructional software might offer.

Concept Instruction

A growing number of programs make an effort to offer conceptual instruction. Some, like the *Fizz & Martina's Math Adventures* series of programs (Tom Snyder Productions) and the *Prime Time Math* series (Tom Snyder Productions), rely on real-world contexts to illustrate mathematical ideas. These are problem-solving situations in which specific concepts are developed in a guided manner to solve the problem.

More common in this category is the use of a visual model and much more directed instruction. *MathKeys* programs (Riverdeep), and the Tenth Planet series (Sunburst) use this approach. In *MathKeys*, the models are set up in a noncontextual manner. The Tenth Planet packages embed models into a contextual format, but these stop short of being real world.

What is most often missing is a way to make the mathematics problem based or to connect the conceptual activity with the symbolic techniques. Furthermore, when students work on a computer, there is little opportunity for discourse, conjecture, or original ideas. Some software even presents concepts in such a fashion as to remove learners from thinking and constructing their own understanding. In some instances, the programs might be best used with the teacher controlling the program on a large display screen with the class. In this way, the teacher can pose questions and entertain discussion that is simply not possible with one student on a computer.

Problem Solving

With the current focus on problem solving, more software publishers purport to teach students to solve problems. The *Fizz & Martina* and *Prime Time Math* series can be included in the problem-solving category. Here the problems are not typical story problems awaiting a computation but more thoughtful stories set in real contexts.

At the other end of the spectrum are programs that offer little more than a large library of typical story problems. Usually, the teacher can control for problem difficulty and the operations to be used. These programs would be more valuable if they offered some conceptual assistance if the student gets the problems incorrect, but that is rarely the case.

Logic problem solving is another variant of problem-solving software. This category includes attribute activities, as in *The Zoombinis Logical Adventure* (The Learning Company/Riverdeep) and *Math Arena* (Sunburst), spatial reasoning, as in *Factory Deluxe* (Sunburst), and number patterns and operation sense, as in *Splish Splash Math* (Sunburst).

Drill

Drill programs give students practice with skills that are assumed to have been taught prior to using the programs. In general, a drill program poses questions that are answered directly or by selecting from a multiple-choice list. Many of these programs are set in arcade formats that make them exciting for students who like that sort of video game, but the format has nothing to do with the practice involved.

Drill programs evaluate responses immediately. How they respond to the first or second incorrect answer is one important distinguishing feature. At one extreme, the answer is simply recorded as wrong. There may be a second or third chance to correct it. At the other extreme, the program may branch to an explanation of the correct response. Others may provide a useful hint or supply a visual model to help with the task. Some programs offer record-keeping features for the teacher to keep track of individual student's progress.

Although computer drill is convenient, there is little evidence to suggest that it is any more effective than non-computer drills. A major advantage is in the formats, which can add motivation to an otherwise boring drill.

Guidelines for Selecting and Using Software

There is so much software for mathematics today. Commercially published software is becoming increasingly expensive. Even though most Internet-based applets are free to use, schools must still provide for Internet access and the appropriate hardware. In either case, it is important to make informed decisions when investing limited resources.

How to Select Software

The most important thing to do before purchasing software is to be well informed about the product and to evaluate its merits in an objective manner.

Gathering Information

One of the best sources of information concerning new software is the review section of the NCTM journals or other journals that you respect. Typically, two or three pieces of software are reviewed each issue, or approximately 20 titles per year. Search through 2 or 3 years of back issues to find software that appeals. Many Web sites offer reviews on both commercially available software and Internet-based applets. The Illuminations Web site by NCTM (http://illuminations.nctm.org) is one such site. Another is The Math Forum at Drexel University at http://mathforum.org. When selecting any computer-based tool or instructional software, it is important to evaluate it appropriately.

Next, try to get a preview copy or at least a demonstration version. The latter may not let you actually interact with the software but rather view a form of commercial on your computer. Many distributors will send their software to you for 30-day approval. If this option is available, take advantage of it. Before purchasing, try the software with kids in the grade that will be using it.

Catalogs are useful for finding prices and titles. They are of limited value for understanding how the software works or even what the content is. A title that says it treats "addition and subtraction" does not tell you if this is basic facts, concepts, contextual problems, or multidigit computation. Drill-and-practice software is rarely distinguished from concept software. Remember, it is the content you are interested in, not the game the students will be playing.

Criteria

Here are some things to think about as you review software before purchasing it or using it in your classroom:

- What does this do better than can be done without the computer? Don't select or use software just to put your students on the computer. Be sure to get past the clever graphics and the games. Focus on what students will be learning.
- How are students likely to be engaged with the *content* (not the bells and whistles)? Remember that student reflective thought is the most significant factor in effective instruction. Is the mathematics presented so that it is problematic for the student?
- How easy is the program to use? Students need not be able to learn the program on their own; you can teach them to use it. But there should not be so much tedium in using the program that attention is diverted from the content or students become frustrated.
- What sort of conceptual information is provided? In drill programs, how are wrong answers handled? Are the models or explanations going to aid in student understanding?
- What controls are provided to the teacher? Are there options that can be turned on and off (e.g., sound, types of feedback or help, levels of difficulty)? Is there a provision for record keeping so that you will know what progress individual students have made?
- Is a manual or online instruction available? What is the quality of the manual or instructions? Minimally, the manual should make it clear how the program is to operate and provide assistance for troubleshooting.
- Is printable material available? What is the quality of this material? Many programs come with extensive off-computer activities, lesson plans, Blackline Masters, and suggestions for ways to use the program with your class.

- What is the nature of the licensing agreement? In the case of purchased software, is a site license or network license available? If you purchase a single-user package, it is not legal to install the software on multiple computers. Note also that many programs that come on a CD must have the CD in the computer to run the program. Internet applets require the computer to be connected to the Internet. Do these constraints fit with your school situation?
- Be sure that the program will run on the computers at your school. The software description should indicate the compatible platform(s) (Windows/Macintosh) and the version of the required operating systems.

Guidelines for Using Software

How software is used in mathematics instruction will vary considerably with the topic, the grade level, and the software itself. The following are offered as considerations that you should keep in mind.

- Software should contribute to the objectives of the lesson or unit. It should not be used as an add-on or substitute for more accessible approaches. Its use should take advantage of what technology can do efficiently and well.
- For individualized or small group use, plan to provide specific instructions for how the software is used, and plan to provide time for students to freely explore or practice using the software.
- Combine software activities with off-computer activities (e.g., collect measurement data in the classroom to enter into a spreadsheet).
- Create a management plan for using the software. This could include a schedule for when the software is used (e.g., during centers, during small group work) and a way to assess the effectiveness of the software use. Although some software programs include a way to keep track of student performance, you may need to rely on other assessment strategies to determine whether the software is being effective at meeting the objectives of the lesson or unit.

Mathematics Education Resources on the Internet

In addition to access to Internet-based software applications, or applets, the World Wide Web is a wellspring of information and resources for both teachers and students interested in mathematics and teaching mathematics.

Instead of using a standard search engine to find mathematics-related information, it is better to have some places to begin. Several good Web sites in different categories will usually provide you with more links to other sites than you will have time to search. One place to get good Web sites is from the NCTM's *News Bulletin*, a newsletter sent free to all members nine times each year. For several years, the *Bulletin* has featured information about useful Web sites. Make it a habit to check these out in each issue.

At the end of the chapter you will find a list of Web-based resources arranged by the type of information the site provides. A brief description accompanies each listing but you are encouraged to check these out yourself as Web sites are frequently modified. The types of resources you can expect to find are described here.

Professional Information

Most professional organizations have established Web sites that provide information about their organizations, conferences, current issues and events, publications, and other matters. Often these sites provide useful links to related resources or information. Periodic visits to these sites are a good way to stay current with what is going on.

Teacher Resources

The Internet is a wonderful source of creative and useful lesson ideas. It is also a good way to find out about materials and software. The sites listed in this category were selected because of the quantity of information that is available on them and because the site addresses are not likely to change or disappear quickly. However, these few suggestions only scratch the surface. A large number of other resources are accessible through these sites.

Applets

Collections of applets are usually found on a single Web site. The sites listed in this category are some that have been found to be the most useful collections across content and grade levels. Selected individual applets for particular content areas are listed at the end of each chapter in Section 2 of this book.

Remember that all applets are not equal, just as all software is not equal. Applets vary in how much freedom they offer the student. Some can be considered pure tool software and others are closer to instructional devices. Some applets come packaged with detailed lessons or investigations and questions for students to answer.

Some applets can be downloaded to your computer so that you do not need to be connected to the Web in order to use them. Even in an off-line mode, however, you will still need a current version of a Web browser such as Netscape Browser, Microsoft Internet Explorer, or Apple's Safari to run them.

Reflections on Chapter 8

Writing to Learn

1. Technology has affected the mathematics curriculum and how it is taught in three ways. Explain each, and give an example to support your explanation. Can you think of examples that are not included in this chapter?
2. Describe some of the benefits of using calculators regularly in the mathematics classroom. Which of these seem to you to be the most compelling? What are some of the arguments against using calculators? Answer each of the arguments against calculators as if you were giving a speech at your PTA meeting or arguing for regular use of calculators before your principal.
3. Aside from the special features of graphing calculators, what are some of the benefits that come simply from having a large display screen and the ability to recall prior statements?
4. What are at least three features of graphing calculators that truly improve the learning of mathematics in the middle grades?
5. Describe what is meant by tool software in mathematics. Describe several types.
6. Describe the three categories of computer instructional software. If you have seen or used examples from these categories, use those in your descriptions.
7. What are some criteria that seem most important to you when selecting software?
8. What kind of information can you expect to find on the Internet?

For Discussion and Exploration

1. Talk with some teachers about their use or nonuse of calculators in the classroom. How do the teachers who use them go about doing so? What are the main reasons for not using them? Read the Technology Principle in *Principles and Standards for School Mathematics.* How do the reasons the teachers you talked with compare to the NCTM position?
2. Among the software kept at your school, find one example of drill software and one of some other form of instructional software for mathematics. Try each and decide how it would be used in your classroom (if at all). Be sure to check the documentation for suggested grade levels.
3. Check out at least three of the Web sites suggested below for teacher resources. Be sure to follow some of the links to other sites. Share at least two good ideas with a friend. Get the friend to do the same.
4. Explore three or four applets from one or more of the sites listed under Applets (see below). Select one and try it with children. Teach a lesson that incorporates the applet as either a teacher tool or student activity.

Recommendations for Further Reading

Crown, W. D. (2003). Using technology to enhance a problem-based approach to teaching: What will and what won't work. In F. K. Lester & R. I. Charles (Eds.), *Teaching mathematics through problem solving: pre-K to 6*. Reston, VA: National Council of Teachers of Mathematics.

Crown offers an overview of both calculators and computers for problem-based mathematics. In his discussion of computers, he describes six categories of educational software. For each category, he explains why that particular type of software is or is not particularly useful in problem-based instruction. Crown's categories of software alone make it worth taking a look at this article.

Hillman, S. L., & Malotka, C. M. (2004). Changing views: Fearless families conquering technology together. *Mathematics Teaching in the Middle School, 10*, 169–173.

The authors offer descriptions of three workshops for parents (including two with students attending) designed to illustrate the power of calculator technology. They used both the TI-Math Explorer and the TI-73.

Masalski, W. J., & Elliott, P. C. (Eds.). (2005). *Technology-supported mathematics learning environments: Sixty-seventh yearbook*. Reston, VA: National Council of Teachers of Mathematics.

An excellent collection of perspectives on the use of technology across the grades by noted authorities and practicing teachers alike. Topics include strategies for effective use of technology, examination of virtual manipulatives for young students, dynamic geometry software, the spreadsheet, and much more. A CD is included to illustrate many of the ideas found in the book plus additional sources.

McGehee, J., & Griffith, L. K. (2004). Technology enhances student learning across the curriculum. *Mathematics Teaching in the Middle School, 9*, 344–349.

Five examples of using technology are explored. Understanding graphs (rate of change), decimals, geometry, measurement, and data analysis are the areas the examples cover. This is a good introduction to the use of technology in any of these domains.

National Council of Teachers of Mathematics. (2002). Learning and teaching mathematics with technology [Focus Issue]. *Teaching Children Mathematics, 8*(6).

In the practical manner of *TCM*, this special issue provides excellent articles on all aspects of technology in the classroom for young children.

Thompson, T., & Sproule, S. (2005). Calculators for students with special needs. *Teaching Children Mathematics, 11*, 391–395.

An excellent argument is made for the use of calculators for students who have learning problems that affect their mathematical skills. A framework or flowchart that is easily

used to make decisions about when to allow calculator use is not only appropriate for special students but also for every child. This short article can blunt the objections raised by the calculator critics.

Online Resources

Suggested Applets and Web Links

Professional Information

National Council of Teachers of Mathematics (NCTM)
www.nctm.org

The NCTM Web site is a must for every elementary teacher and teacher of mathematics. It includes specific information for teachers, parents, leaders, and researchers. The home page changes almost monthly, providing up-to-date information about conferences, publications, news, and more. The site also provides a mechanism for joining the council, registering for conferences, purchasing publications and products, and linking to the Illuminations site (see separate entry). Members can access their journals online, subscribe to a special electronic journal, and renew memberships. You can choose to receive a monthly e-mail update informing you of recent additions to the Web site.

Eisenhower National Clearinghouse (ENC)
www.enc.org

This is a site of links to information about standards-based curriculum materials for K–12 mathematics and science, information and articles on standards-based issues, information about TIMSS, and ideas for classroom use. Look especially at ENC Features.

American Association for the Advancement of Science
www.aaas.org

This is the association for Project 2061, a national science standards document. Access to this and other information concerning science education is available.

Association for Supervision and Curriculum Development (ACSD)
www.ascd.org

ASCD is an international nonprofit educational association that is committed to successful teaching and learning for all.

EQUALS and Family Math
http://equals.lhs.berkeley.edu

The EQUALS program is dedicated to equal opportunity for females in mathematics and science. It publishes Family Math and other valuable resources. For those interested in issues surrounding gender equity and parental involvement, this is a good place to begin.

Mathematically Sane
www.mathematicallysane.com

This is an important and unique Web site that provides evidence for and discussion of the success of standards-based initiatives in mathematics education. Its mission statement identifies the site as created by a "grassroots organization of teachers, administrators, teacher educators, parents, and mathematicians concerned about the future of mathematics education."

Teacher Resources

NCTM Illuminations
http://illuminations.nctm.org

This is an incredible site developed by NCTM to provide Internet resources for teaching and learning intended to "illuminate" *Principles and Standards for School Mathematics.* You can find resources from lesson ideas to "math-lets" (applets designed to provide tools for developing understanding in mathematics). Also at this site are multimedia investigations for students and links to video vignettes designed to promote professional reflection.

The Math Forum
http://mathforum.org

Along with the NCTM sites, this may be your most important source of information and links to useful sites. The forum has resources for both teachers and students. There are suggestions for lessons, puzzles, and activities, plus links to other sites with similar information. There are forums where teachers can talk with other teachers. Two pages accept questions about mathematics from students or teachers (Ask Dr. Math) and about teaching mathematics from teachers (Teacher 2 Teacher). Problems are regularly posted, and solutions can be entered via the Internet. Information is available about software, and some can be downloaded free.

MegaMath!
www.c3.lanl.gov/mega-math

This is a great site! A project of the Computer Research and Applications Group at Los Alamos National Laboratory, it is intended to bring unusual and important mathematical ideas to elementary classrooms.

Math Archives: K–12 Internet Sites
http://archives.math.utk.edu/k12.html

This page contains a very large collection of links to lessons, software, information (both public domain and commercial), and curriculum materials. Each of the hundreds of sites has a brief description. If you are looking for an Internet address for a mathematics resource, this may be a good place to look.

EdWeb Home Page
www.edwebproject.org

This site explores issues of educational reform and contains links to online resources around the world.

Home Page for New Math Teachers
http://people.clarityconnect.com/webpages/terri/terri.html

A veteran teacher, Terri Husted, offers advice on a list of subjects challenging the new teacher. In addition, she provides clever problems to pose to students, links to a huge number of useful sites, and much more.

Annenberg/CPB Projects
www.learner.org

The site lists free online learning activities, including information about all sorts of interesting uses of mathematics and science in the real world, resources for free and inexpensive materials from Annenberg, and information about funding opportunities. It is a tremendous resource.

Center for Implementing Technology in Education (CITEd): Math Matrix
www.citeducation.org/mathmatrix/default.asp
CITEd's Math Matrix is a small but useful database of technology products that supports instruction in mathematics for students with special needs. Each product evaluation includes a link to the supplier's Web site.

U.S. Census Data
www.census.gov
Copious statistical information by state, county, or voting district.

The World Fact Book
www.odci.gov/cia/publications/factbook/index.html
This page provides demographic information for every nation in the world, including population, age distributions, death and birth rates, and information on the economy, government, transportation, and geography. Maps are included as well.

Applets

National Council of Teachers of Mathematics e-Examples
http://standards.nctm.org/document/eexamples/index.htm
Many of these applets are referenced in and directly support the text of *Principles and Standards for School Mathematics.* They are also available on the CD version of the Standards. Most are also available on the *Illuminations* site.

NCTM Illuminations
http://illuminations.nctm.org
Check both the i-Math Investigations (interactive math lessons, most built around applets) and Interactive Mathlets (a collection of applets). The Math-let applications cover the K–12 spectrum. They are ordered alphabetically, so be sure to check out the full list. This is a good collection of quality tools. The i-Math Investigations include all of the applets from the e-examples.

The National Library for Virtual Manipulatives (NLVM)
http://matti.usu.edu/nlvm/nav/vlibrary.html
This NSF-funded site is located at Utah State University. It contains a huge collection of applets organized by the five content strands of the *Standards* and also by the same four grade bands. You may not find all of these applets equally interesting or usable but there is a lot more to like than not. Give the site a thorough look.

Arcytech
http://arcytech.org/java
At the time this was written this site included tool applets for base-ten blocks, pattern blocks, Cuisenaire rods, fraction bars, and integer bars. There is also an extended interactive lesson developing the Pythagorean theorem.

Shodor Interactivate (Shodor Education Foundation)
http//www.shodor.org/interactive
The site contains a huge list of applets that continues to grow. In addition, there are lessons and activities, many that include the use of applets. Applets (referred to as "activities") are arranged by content rather than grade level, so be sure to look through the full list. This is a valuable site, especially for teachers in the upper grades and middle school.

Count On
www.mathsyear2000.org
This site is sponsored by the National Grid for Learning and contains all sorts of resources in addition to applets for mathematics. Click on the Explorer button to find a number of applets as well as other useful features such as a mathematics dictionary. Check out other areas of this site as well.

An additional list of books and articles related to the ideas in this chapter can be found on the Companion Web site at **www.ablongman.com/vandewalle6e.**