Modeling Derivatives Applications in Matlab, C++, and Excel
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Modeling Derivatives Applications in Matlab, C++, and Excel

Justin London
To the memory of my grandparents, Milton and Evelyn London; my parents, Leon and Leslie; and my sister, Joanna.
CONTENTS

Preface xv
Acknowledgments xix
About the Author xxi

1 SWAPS AND FIXED INCOME INSTRUMENTS 1
1.1 Eurodollar Futures 2
1.2 Treasury Bills and Bonds 3
   Hedging with T-Bill Futures 6
   Long Futures Hedge: Hedging Synthetic Futures on 182-Day T-Bill 7
1.3 Computing Treasury Bill Prices and Yields in Matlab 10
1.4 Hedging Debt Positions 11
   Hedging a Future 91-Day T-Bill Investment with T-Bill Call 11
   Short Hedge: Managing the Maturity Gap 12
   Maturity Gap and the Carrying Cost Model 14
   Managing the Maturity Gap with Eurodollar Put 14
   Short Hedge: Hedging a Variable-Rate Loan 15
1.5 Bond and Swap Duration, Modified Duration, and DV01 18
   Hedging Bond Portfolios 20
1.6 Term Structure of Rates 24
1.7 Bootstrap Method 25
1.8 Bootstrapping in Matlab 28
1.9 Bootstrapping in Excel 30
1.10 General Swap Pricing in Matlab 33
   Description 43
1.11 Swap Pricing in Matlab Using Term Structure Analysis 45
1.12 Swap Valuation in C++ 50
1.13 Bermudan Swaption Pricing in Matlab 61
Endnotes 65

vii
## Contents

### 2 COPULA FUNCTIONS

2.1 Definition and Basic Properties of Copula Functions

2.2 Classes of Copula Functions
   - Multivariate Gaussian Copula
   - Multivariate Student’s $T$ Copula

2.3 Archimedean Copulae

2.4 Calibrating Copulae
   - Exact Maximum Likelihood Method (EML)
   - The Inference Functions for Margins Method (IFM)
   - The Canonical Maximum Likelihood Method (CML)

2.5 Numerical Results for Calibrating Real-Market Data
   - Bouyè, Durrelman, Nikeghbali, Riboulet, and Roncalli Method
   - Mashal and Zeevi Method

2.6 Using Copulas in Excel

Endnotes

### 3 MORTGAGE-BACKED SECURITIES

3.1 Prepayment Models

3.2 Numerical Example of Prepayment Model

3.3 MBS Pricing and Quoting

3.4 Prepayment Risk and Average Life of MBS

3.5 MBS Pricing Using Monte Carlo in C++

3.6 Matlab Fixed-Income Toolkit for MBS Valuation

3.7 Collateralized Mortgage Obligations (CMOs)

3.8 CMO Implementation in C++

3.9 Planned Amortization Classes (PACS)

3.10 Principal- and Interest-Only Strips

3.11 Interest Rate Risk

3.12 Dynamic Hedging of MBS
   - The Multivariable Density Estimation Method

Endnotes

### 4 COLLATERALIZED DEBT OBLIGATIONS

4.1 Structure of CDOs
   - Cash Flow CDOs
   - Market Value CDOs
   - Balance Sheet Cash Flows CDOs
   - Arbitrage CDOs
   - Arbitrage Market Value CDOs
## Contents

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arbitrage Cash Flow CDOs</td>
<td>167</td>
</tr>
<tr>
<td>Credit Enhancement in Cash Flow Transactions</td>
<td>167</td>
</tr>
<tr>
<td>Credit Enhancement in Market Value Transactions: Advance Rates and the Over-Collateralization Test</td>
<td>167</td>
</tr>
<tr>
<td>Minimum Net Worth Test</td>
<td>169</td>
</tr>
<tr>
<td>Transaction Characteristics</td>
<td>171</td>
</tr>
<tr>
<td>4.2 Synthetic CDOs</td>
<td>171</td>
</tr>
<tr>
<td>Fully Funded Synthetic CDOs</td>
<td>177</td>
</tr>
<tr>
<td>Partially and Unfunded Synthetic CDOs</td>
<td>179</td>
</tr>
<tr>
<td>4.3 Balance Sheet Management with CDS</td>
<td>181</td>
</tr>
<tr>
<td>4.4 The Distribution of Default Losses on a Portfolio</td>
<td>181</td>
</tr>
<tr>
<td>4.5 CDO Equity Tranche</td>
<td>186</td>
</tr>
<tr>
<td>CDO Equity Tranche Performance</td>
<td>186</td>
</tr>
<tr>
<td>The CDO Embedded Option</td>
<td>187</td>
</tr>
<tr>
<td>The Price of Equity</td>
<td>188</td>
</tr>
<tr>
<td>Using Moody’s Binomial Expansion Technique to Structure Synthetic CDOs</td>
<td>189</td>
</tr>
<tr>
<td>Correlation Risk of CDO Tranches</td>
<td>193</td>
</tr>
<tr>
<td>4.6 CDO Tranche Pricing</td>
<td>196</td>
</tr>
<tr>
<td>4.7 Pricing Equation</td>
<td>197</td>
</tr>
<tr>
<td>4.8 Simulation Algorithm</td>
<td>197</td>
</tr>
<tr>
<td>4.9 CDO Pricing in Matlab</td>
<td>199</td>
</tr>
<tr>
<td>4.10 CDO Pricing in C++</td>
<td>208</td>
</tr>
<tr>
<td>4.11 CDO^2 Pricing</td>
<td>216</td>
</tr>
<tr>
<td>4.12 Fast Loss Calculation for CDOs and CDO^2s</td>
<td>216</td>
</tr>
<tr>
<td>Fast Algorithm for Computing CDO Tranche Loss in Matlab</td>
<td>218</td>
</tr>
<tr>
<td>Endnotes</td>
<td>220</td>
</tr>
</tbody>
</table>

## 5 CREDIT DERIVATIVES

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.1 Credit Default Swaps</td>
<td>224</td>
</tr>
<tr>
<td>5.2 CDS Day Counting Conventions</td>
<td>226</td>
</tr>
<tr>
<td>5.3 General Valuation of Credit Default Swaps</td>
<td>226</td>
</tr>
<tr>
<td>5.4 Hazard Rate Function</td>
<td>228</td>
</tr>
<tr>
<td>5.5 Poisson and Cox Processes</td>
<td>229</td>
</tr>
<tr>
<td>5.6 Valuation Using a Deterministic Intensity Model</td>
<td>232</td>
</tr>
<tr>
<td>5.7 Hazard Rate Function Calibration</td>
<td>235</td>
</tr>
<tr>
<td>5.8 Credit Curve Construction and Calibration</td>
<td>248</td>
</tr>
</tbody>
</table>
Contents

5.9 Credit Basket Default Swaps Pricing  249
  Generation of Correlated Default Stopping Times  250
  Sampling from Elliptical Copulae  250
  The Distribution of Default Arrival Times  252
  Basket CDS Pricing Algorithm  252
5.10 Credit Basket Pricing in Matlab  255
5.11 Credit Basket Pricing in C++  264
5.12 Credit Linked Notes (CLNs)
  CLNs with Collateralized Loan or Bond Obligations (CLOs or CBOs)  295
  Pricing Tranchmed Credit Linked Notes  296
  Regulatory Capital  296
Endnotes  297

6 WEATHER DERIVATIVES  299
6.1 Weather Derivatives Market  300
6.2 Weather Contracts
  CME Weather Futures  303
6.3 Modeling Temperature
  Noise Process  308
  Mean-Reversion  309
6.4 Parameter Estimation  310
6.5 Volatility Estimation  310
6.6 Mean-Reversion Parameter Estimation  311
6.7 Pricing Weather Derivatives
  Model Framework  312
  Pricing a Heating Degree Day Option  313
6.8 Historical Burn Analysis  316
6.9 Time-Series Weather Forecasting  318
6.10 Pricing Weather Options in C++  328
Endnotes  330

7 ENERGY AND POWER DERIVATIVES  333
7.1 Electricity Markets  334
7.2 Electricity Pricing Models
  Modeling the Price Process  336
  One-Factor Model  337
  Estimating the Deterministic Component  340
  Estimation of the Stochastic Process for the One-Factor Models  341
  Two-Factor Model  342
## Contents

7.3 Swing Options 344
7.4 The Longstaff-Schwartz Algorithm for American and Bermudan Options 345
   The LSM Algorithm 346
7.5 Extension of Longstaff-Schwartz to Swing Options 348
7.6 General Case: Upswings, Downswings, and Penalty Functions 351
7.7 Swing Option Pricing in Matlab 352
7.8 LSM Simulation Results 352
   Upper and Lower Boundaries 354
   Exercise Strategies 356
   The Threshold of Early Exercise 358
   Interplay Between Early Exercise and Option Value 360
7.9 Pricing of Energy Commodity Derivatives 362
   Cross-Commodity Spread Options 362
   Model 1 364
   Model 2 365
   Model 3 366
7.10 Jump Diffusion Pricing Models 368
   Model 1a: Affine Mean-Reverting Jump-Diffusion Process 368
   Model 1b 369
   Model 2a: Time-Varying Drift Component 370
   Model 2b: Time-Varying Version of Model 1b 372
7.11 Stochastic Volatility Pricing Models 372
   Model 3a: Two-Factor Jump-Diffusion Affine Process with Stochastic Volatility 372
7.12 Model Parameter Estimation 373
   ML-CCF Estimators 375
   ML-MCCF Estimators 376
   Spectral GMM Estimators 379
   Simulation 383
7.13 Parameter Estimation in Matlab 385
7.14 Energy Commodity Models 385
7.15 Natural Gas 387
   Natural Gas Markets 387
   Natural Gas Spot Prices 389
7.16 Gas Pricing Models 390
   One-Factor Model 390
   Two-Factor Model 391
   Calibration 393
### 8 PRICING POWER DERIVATIVES: THEORY AND MATLAB IMPLEMENTATION

8.1 Introduction 407  
8.2 Power Markets 409  
8.3 Traditional Valuation Approaches Are Problematic for Power 410  
8.4 Fundamentals-Based Models 413  
8.5 The PJ Model—Overview 415  
8.6 Model Calibration 419  
8.7 Using the Calibrated Model to Price Options 423  
  - Daily Strike Options 423  
  - Monthly Strike Options 423  
  - Spark Spread Options 424  
8.8 Option Valuation Methodology 424  
  - Splitting the (Finite) Difference: Daily Strike and Monthly Strike Options 424  
  - Matlab Implementation for a Monthly Strike Option 426  
  - Spark Spread Options 434  
  - Matlab Implementation of Spark Spread Option Valuation 434  
8.9 Results 439  
8.10 Summary 443  
Endnotes 443  
References 445

### 9 COMMERCIAL REAL ESTATE ASSET-BACKED SECURITIES

9.1 Introduction 447  
9.2 Motivations for Asset-Backed Securitization 449  
9.3 Concepts of Securitizing Real Estate Cash Flows 450  
9.4 Commercial Real Estate-Backed Securitization (CREBS)—Singapore’s Experience 452  
9.5 Structure of a Typical CREBS 456  
  - A CREBS Case by Visor Limited 457
## Contents

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.6 Pricing of CREBS</td>
<td>459</td>
</tr>
<tr>
<td>Swaps and Swaptions</td>
<td>459</td>
</tr>
<tr>
<td>The Cash Flow Swap Structure for CREBS</td>
<td>459</td>
</tr>
<tr>
<td>9.7 Valuation of CREBS Using a Swap Framework</td>
<td>460</td>
</tr>
<tr>
<td>Basic Swap Valuation Framework</td>
<td>460</td>
</tr>
<tr>
<td>Pricing of Credit Risks for CREBS Using the Proposed Swap Model</td>
<td>461</td>
</tr>
<tr>
<td>Modeling Default Risks in the CREBS Swap</td>
<td>461</td>
</tr>
<tr>
<td>9.8 Numerical Analysis of Default Risks for a Typical CREBS</td>
<td>463</td>
</tr>
<tr>
<td>Monte Carlo Simulation Process</td>
<td>463</td>
</tr>
<tr>
<td>Input Parameters</td>
<td>464</td>
</tr>
<tr>
<td>Analysis of Results</td>
<td>465</td>
</tr>
<tr>
<td>9.9 Matlab Code for the Numerical Analysis</td>
<td>467</td>
</tr>
<tr>
<td>9.10 Summary</td>
<td>470</td>
</tr>
<tr>
<td>Endnotes</td>
<td>470</td>
</tr>
</tbody>
</table>

**A INTEREST RATE TREE MODELING IN MATLAB**

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>A.1 BDT Modeling in Matlab</td>
<td>473</td>
</tr>
<tr>
<td>A.2 Hull-White Trees in Matlab</td>
<td>478</td>
</tr>
<tr>
<td>A.3 Black-Karasinski Trees in Matlab</td>
<td>486</td>
</tr>
<tr>
<td>A.4 HJM Pricing in Matlab</td>
<td>490</td>
</tr>
<tr>
<td>Description</td>
<td>492</td>
</tr>
<tr>
<td>Syntax</td>
<td>493</td>
</tr>
<tr>
<td>Arguments</td>
<td>493</td>
</tr>
<tr>
<td>Examples</td>
<td>493</td>
</tr>
<tr>
<td>Creating an HJM Volatility and Pricing Model</td>
<td>494</td>
</tr>
<tr>
<td>A.5 Matlab Excel Link Example</td>
<td>497</td>
</tr>
<tr>
<td>A.6 Two-Factor HJM Model Implementation in Matlab</td>
<td>500</td>
</tr>
<tr>
<td>Endnotes</td>
<td>501</td>
</tr>
</tbody>
</table>

**B CHAPTER 7 CODE**

**REFERENCES**

**INDEX**
PREFACE

Given the explosive growth in new financial derivatives such as credit derivatives, hundreds of financial institutions now market these complex instruments and employ thousands of financial and technical professionals needed to model them accurately and effectively. Moreover, the implementation of these models in C++ and Matlab (two widely used languages for implementing and building derivatives models) has made programming skills in these languages important for practitioners to have. In addition, the use of Excel is also important as many trading desks use Excel as a front-end trading application.

*Modeling Derivatives Applications in Matlab, C++, and Excel* is the first book to cover in detail important derivative pricing models for credit derivatives (for example, credit default swaps and credit-linked notes), collateralized-debt obligations (CDOs), mortgage-backed securities (MBSs), asset-backed securities (ABSs), swaps, fixed income securities, and increasingly important weather, power, and energy derivatives using Matlab, C++, and Excel. Readers will benefit from both the mathematical derivations of the models, the theory underlying the models, as well as the code implementations.

Throughout this book, numerous examples are given using Matlab, C++, and Excel. Examples using actual real-time Bloomberg data show how these models work in practice. The purpose of the book is to teach readers how to properly develop and implement derivatives applications so that they can adapt the code for their own use as they develop their own applications. The best way to learn is to follow the examples and run the code. The chapters cover the following topics:

- Chapter 1: Swaps and fixed income securities
- Chapter 2: Copulas and copula methodologies
- Chapter 3: Mortgage-backed securities
- Chapter 4: Collateralized-debt obligations
- Chapter 5: Credit derivatives
- Chapter 6: Weather derivatives
- Chapter 7: Energy and power derivatives
- Chapter 8: Also covers model implementations for energy derivatives using Matlab, but is written and based on the proprietary work of its author, Craig Pirrong, professor of finance and director of the Global Energy Management Institute at the University of Houston.
Preface

- Chapter 9: Commercial real-estate backed securities (a type of asset-backed security), which is written and is based on the proprietary work of its author, Tien-Foo Sing, professor in the Department of Real Estate Finance at the National University of Singapore.

In order to provide different perspectives to readers and provide as much useful information as possible, the work and models developed and written by various leading practitioners and experts for certain topics are provided and incorporated throughout the book. Thus, not only does this book cover complex derivatives models and provide all of the code (which can be downloaded using a secure ID code from the companion Web site at www.ftpress.com/title/0131962590), but it also incorporates important work contributions from leading practitioners in the industry. For instance, the work of Galiani (2003) is discussed in the chapter on copulas and credit derivatives. The work of Picone (2004) is discussed in the chapter on collateralized-debt obligations. The work of Johnson (2004) is discussed in the chapters on fixed-income instruments and mortgage-backed securities. The valuable work for energy derivatives of Doerr (2002), Xiang (2004), and Xu (2004) is given. In Chapter 8, Craig Pirrong discusses the Pirrong-Jermayakan model, a two-dimensional alternating implicit difference (ADI) finite difference scheme for pricing energy derivatives. In Chapter 9, Tien-Foo Sing discusses using Monte Carlo to price asset-based securities. Moreover, numerous individuals named in the acknowledgments contributed useful code throughout the book.

The book emphasizes how to implement and code complex models for pricing, trading, and hedging using C++, Matlab, and Excel. The book does not focus on design patterns or best coding practices (these issues may be discussed in subsequent editions of the book.) Efficiency and modularity are important design goals in building robust object-oriented code. In some cases in this book, the C++ code provided could perhaps be more modular as with some of the routines in building interest rate trees. The emphasis throughout the book has been to provide working implementations for the reader to adapt. However, the book does provide some discussions and helpful tips for building efficient models. For instance, memory allocation for data structures is always an issue when developing a model that requires use and storage of multi-dimensional data. Use of a predefined two-dimensional array, for instance, is not the most efficient way to allocate memory since it is fixed in size. A lot of memory may be unutilized and wasted if you do not know how large the structure needs to be to store the actual data. On the other hand, the predefined array sizes may turn out not to be large enough.

Although two-dimensional arrays are easy to define, use of array template classes (that can handle multiple dimensions) and vectors (of vectors) in the Standard Template Library in C++ are more efficient because they are dynamic and only use as much memory as is needed. Such structures are used in the book, although some two-dimensional arrays are used as well. Matlab, a matrix manipulation language, provides automatic memory allocation of memory as data is used if no array sizes are predefined. All data in Matlab are treated as matrix objects; e.g., a single number is treated as a 1 x 1 array. Data can be added or removed from an object and the object will dynamically expand or reduce the amount of memory space as needed.
Preface

While every effort has been made to catch all typos and errors in the book, inevitably in a book of this length and complexity, there may still be a few. Any corrections will be posted on the Web site.

Hopefully, this book will give you the foundation to develop, build, and test your own models while saving you a great deal of development time through use of pre-tested robust code.

Supplemental Files on the Web Site

To download the code in this book, you must first register online. You will need a valid email address and the access code that is printed inside the envelope located at the back of the book.

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Justin London has developed fixed-income and equity models for trading companies and his own quantitative consulting firm. He has analyzed and managed bank corporate loan portfolios using credit derivatives in the Asset Portfolio Group of a large bank in Chicago, Illinois, as well as advised several banks in their implementation of derivative trading systems. London is the founder of a global online trading and financial technology company. A graduate of the University of Michigan, London holds a B.A. in economics and mathematics, an M.A. in applied economics, and an M.S. in financial engineering, computer science, and mathematics, respectively.