Chapter 1

Teaching Mathematics in the Era of the NCTM Standards

In this changing world, those who understand and can do mathematics will have significantly enhanced opportunities and options for shaping their futures. Mathematical competence opens doors to productive futures. A lack of mathematical competence keeps those doors closed. . . . All students should have the opportunity and the support necessary to learn significant mathematics with depth and understanding. There is no conflict between equity and excellence.

NCTM (2000, p. 50)

Someday soon you will find yourself in front of a class of students, or perhaps you are already teaching. What general ideas will guide the way you will teach mathematics? This book will help you become comfortable with the mathematics content of the pre-K–8 curriculum. You will also learn about research-based strategies for helping children come to know mathematics and be confident in their ability to do mathematics. These two things—your knowledge of mathematics and how students learn mathematics—are the most important tools you can acquire to be an effective teacher of mathematics. However, outside influences and research will affect the mathematics teaching in your classroom as well.

For at least two decades, mathematics education has been undergoing slow but steady changes. The impetus for these changes, in both the content of school mathematics and the way mathematics is taught, can be traced to various sources, including knowledge gained from research. One significant factor in this change has been the professional leadership of the National Council of Teachers of Mathematics (NCTM), an organization of teachers and mathematics educators. Another factor is the public or political pressure for change in mathematics education due largely to less-than-stellar U.S. student performance in international studies. In reaction, state standards and the No Child Left Behind Act (NCLB) press for higher levels of achievement, more testing, and increased teacher accountability. The reform agendas of NCTM and those of the political sector often seem to press teachers in different directions. Although high expectations for students are important, testing alone is not an appropriate approach to improved student learning. According to NCTM, "Learning mathematics is maximized when teachers focus on mathematical thinking and reasoning" (www.nctm.org). The views of NCTM are clearly reflected in the ideas discussed in this book.

As you prepare to help children learn mathematics, it is important to have some perspective on the forces that affect change in the mathematics classroom. This chapter addresses the leadership that NCTM provides for mathematics education and also the major pressures on mathematics education from outside influences.

Ultimately, it is you, the teacher, who will shape mathematics for the children you teach. Your beliefs about what it means to know and do mathematics and about how children come to make sense of mathematics will affect how you approach instruction. These beliefs will undoubtedly be affected, directly or indirectly, by the significant ideas on mathematics education that you will read about in this chapter.

The National Standards-Based Movement

In April 2000, the National Council of Teachers of Mathematics (NCTM) released *Principles and Standards for School Mathematics*, an update of its original standards document released 11 years earlier in 1989. With this most important document, the council continues to guide a revolutionary reform movement in mathematics education, not just in the United States and Canada but also throughout the world.

The momentum for reform in mathematics education began in the early 1980s in response to a "back to basics" movement that emphasized "reading, writing, and arithmetic." As a result, problem solving became an important strand in the mathematics curriculum. The work of Jean

Piaget and other developmental psychologists helped to focus research on how children can best learn mathematics.

This momentum came to a head in 1989, when NCTM published *Curriculum and Evaluation Standards for School Mathematics* and the standards movement or reform era in mathematics education began. It continues today. No other document has ever had such an enormous effect on school mathematics or on any other area of the curriculum. In 1991, NCTM published *Professional Standards for Teaching Mathematics*. The *Professional Standards* and the companion document *Mathematics Teaching Today* articulate a vision of teaching mathematics and build on the notion found in the *Curriculum Standards* that significant mathematics achievement is a vision for all children, not just a few. NCTM completed the package with the *Assessment Standards for School Mathematics* in 1995 (see Chapter 5). The *Assessment Standards* shows clearly the necessity of integrating assessment with instruction and indicates the key role that assessment plays in implementing change.

From 1989 to 2000, these three documents guided the reform movement in mathematics education, directly leading in 2000 to the publication of *Principles and Standards for School Mathematics*, which is an update of all three original standards documents and further articulates the ideals, processes, and content that should be emphasized in pre-K through grade 12 classrooms and programs. In 2006, NCTM released *Curriculum Focal Points*, a little publication with a big message—mathematics at each grade level needs to focus, go into more depth, and show connections. With continued guidance from NCTM and the sustained hard work of teachers and mathematics educators at all levels, mathematics teaching and learning will continue to improve and move the country forward to a curriculum that is more challenging and meaningful to students. In the following sections, we discuss these documents, especially the *Principles and Standards*, as well as other reports, because their message is critical to your work as a mathematics teacher.

Principles and Standards for School Mathematics

Principles and Standards for School Mathematics (2000) is designed to provide guidance and direction for teachers and other leaders in pre-K–12 mathematics education. After almost 10 years, *Principles and Standards* remains the most significant reference for these educators on mathematical knowledge. While it is important that teachers read and reflect on the actual document, the next few pages will provide you with an idea of what you will find there.

The Six Principles

One of the most important features of *Principles and Standards for School Mathematics* is the articulation of six principles fundamental to high-quality mathematics education:

- Equity
- Curriculum
- Teaching
- Learning
- Assessment
- Technology

According to *Principles and Standards*, these principles must be "deeply intertwined with school mathematics programs" (NCTM, 2000, p. 12). The principles make it clear that excellence in mathematics education involves much more than simply listing content objectives.

The Equity Principle

> Excellence in mathematics education requires equity—high expectations and strong support for all students. (NCTM, 2000, p. 12)

The strong message of the Equity Principle is high expectations for all students. All students must have the opportunity and adequate support to learn mathematics "regardless of personal characteristics, backgrounds, or physical challenges" (p. 12). The message of high expectations for all is interwoven throughout the document as a whole.

The Curriculum Principle

> A curriculum is more than a collection of activities: it must be coherent, focused on important mathematics, and well articulated across the grades. (NCTM, 2000, p. 14)

Coherence speaks to the importance of building instruction around "big ideas" both in the curriculum and in daily classroom instruction. Students must be helped to see that mathematics is an integrated whole, not a collection of isolated bits and pieces.

Mathematical ideas are "important" if they help in the development of other ideas, link ideas one to another, or serve to illustrate the discipline of mathematics as a human endeavor.

The Teaching Principle

> Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well. (NCTM, 2000, p. 16)

What students learn about mathematics almost entirely depends on the experiences that teachers provide every day in the classroom. To provide high-quality mathematics education, teachers must (1) understand deeply the mathematics they are teaching; (2) understand how children learn mathematics, including a keen awareness of the individual mathematical development of their own students; and (3) select instructional tasks and strategies that will enhance learning.

"Teachers' actions are what encourage students to think, question, solve problems, and discuss their ideas, strategies, and solutions" (p. 18).

The Learning Principle

> Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge. (NCTM, 2000, p. 20)

The learning principle is based on two fundamental ideas. First, learning mathematics with understanding is essential. Mathematics today requires not only computational skills but also the ability to think and reason mathematically in order to solve the new problems and learn the new ideas that students will face in the future.

Second, the principle states quite clearly that students *can* learn mathematics with understanding. Learning is enhanced in classrooms where students are required to evaluate their own ideas and those of others, are encouraged to make mathematical conjectures and test them, and are helped to develop their reasoning skills.

The Assessment Principle

> Assessment should support the learning of important mathematics and furnish useful information to both teachers and students. (NCTM, 2000, p. 22)

In the authors' words, "Assessment should not merely be done *to* students; rather, it should also be done *for* students, to guide and enhance their learning" (p. 22). Ongoing assessment highlights for students the most important mathematics concepts. Assessment that includes ongoing observation and student interaction encourages students to articulate and, thus, clarify their ideas. Feedback from daily assessment helps students establish goals and become more independent learners.

Assessment should also be a major factor in making instructional decisions. By continuously gathering information about student growth and understanding, teachers can better make the daily decisions that support student learning. For assessment to be effective, teachers must use a variety of assessment techniques, understand their mathematical goals deeply, and have a good idea of how their students may be thinking about or misunderstanding the mathematics that is being developed.

The Technology Principle

> Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning. (NCTM, 2000, p. 24)

Calculators, computers, and other technologies should be seen as essential tools for doing and learning mathematics in the classroom. Technology permits students to focus on mathematical ideas, to reason, and to solve problems in ways that are often impossible without these tools. Technology enhances the learning of mathematics by allowing for increased exploration and enhanced representation of ideas. It extends the range of problems that can be accessed.

The Five Content Standards

Principles and Standards includes four grade bands: pre-K–2, 3–5, 6–8, and 9–12. The new emphasis on preschool recognizes the need to highlight the critical years before children enter kindergarten. Rather than use different sets of mathematical topics for each grade band, the authors agreed on a common set of five content standards throughout the grades (see Appendix A). Section 2 of this book (Chapters 8 through 23) is devoted to elaborating on each of the content standards listed below:

- Number and Operations
- Algebra
- Geometry
- Measurement
- Data Analysis and Probability

Each content standard includes a small set of goals applicable to all grade bands. Then, each grade-band chapter provides specific expectations for what students should know. These grade-band expectations are also concisely listed in the appendix to the *Standards* and in Appendix A of this book.

Pause and Reflect

Pause now and turn to Appendix A. Spend a few minutes with these expectations for the grade band in which you are most interested. How do these expectations compare with the mathematics you experienced when you were in school?

Although the same five content standards apply across all grades, you should not infer that each strand has equal weight or emphasis in every grade band. Number and Operations is the most heavily emphasized strand from pre-K through grade 5 and continues to be important in the middle grades, with a lesser emphasis in grades 9–12. That same emphasis is reflected in this book, with Chapters 8 to 13 and 15 to 18 addressing content found in the Number and Operations standard.

Algebra is clearly intended as a strand for all grades. This was likely not the case when you were in school. Today, most states and provinces include algebra objectives at every grade level. In this book, Chapter 14 addresses this strand.

Note that Geometry and Measurement are separate strands, suggesting the unique importance of each of these two areas to the elementary and middle grades curriculum.

The Five Process Standards

Following the five content standards, *Principles and Standards* lists five process standards:

- Problem Solving
- Reasoning and Proof

- Communication
- Connections
- Representation

The process standards refer to the mathematical process through which students should acquire and use mathematical knowledge. The statement of the five process standards can be found in Table 1.1.

These five processes should not be regarded as separate content or strands in the mathematics curriculum. Rather, they direct the methods or processes of doing all mathematics and, therefore, should be seen as integral components of all mathematics learning and teaching.

To teach in a way that reflects these process standards is one of the best definitions of what it means to teach "according to the *Standards*."

The **Problem Solving** standard clearly views problem solving as the vehicle through which children develop mathematical ideas. Learning and doing mathematics *as you solve problems* is probably the most significant difference in the *Standards* approach versus previous methodologies.

If problem solving is the focus of mathematics, the **Reasoning and Proof** standard emphasizes the logical thinking that helps us decide if and why our answers make sense. Students need to develop the habit of providing a rationale as an integral part of every answer. It is essential for students to learn the value of justifying ideas through logical argument.

The **Communication** standard points to the importance of being able to talk about, write about, describe, and explain mathematical ideas. Learning to communicate in mathematics fosters interaction and exploration of ideas in the classroom as students learn in an active, verbal environment. No better way exists for wrestling with or cementing an idea than attempting to articulate it to others.

The **Connections** standard has two separate thrusts. First, it refers to connections within and among mathematical ideas. For example, fractional parts of a whole are connected to concepts of decimals and percents. Students need opportunities to see how mathematical ideas build on one another in a useful network of connected ideas.

Second, mathematics should be connected to the real world and to other disciplines. Children should see that mathematics plays a significant role in art, science, language arts, and social studies. This suggests that mathematics should frequently be integrated with other discipline areas and that applications of mathematics in the real world should be explored.

The **Representation** standard emphasizes the use of symbols, charts, graphs, manipulatives, and diagrams as powerful methods of expressing mathematical ideas and relationships. Symbolism in mathematics, along with visual aids such as charts and graphs, should be understood by students as ways of communicating mathematical ideas to other people. Moving from one representation to another

Table 1.1

The Five Process Standards from *Principles and Standards for School Mathematics*

Problem Solving Standard Instructional programs from prekindergarten through grade 12 should enable all students to—	• Build new mathematical knowledge through problem solving • Solve problems that arise in mathematics and in other contexts • Apply and adapt a variety of appropriate strategies to solve problems • Monitor and reflect on the process of mathematical problem solving
Reasoning and Proof Standard Instructional programs from prekindergarten through grade 12 should enable all students to—	• Recognize reasoning and proof as fundamental aspects of mathematics • Make and investigate mathematical conjectures • Develop and evaluate mathematical arguments and proofs • Select and use various types of reasoning and methods of proof
Communication Standard Instructional programs from prekindergarten through grade 12 should enable all students to—	• Organize and consolidate their mathematical thinking through communication • Communicate their mathematical thinking coherently and clearly to peers, teachers, and others • Analyze and evaluate the mathematical thinking and strategies of others • Use the language of mathematics to express mathematical ideas precisely
Connections Standard Instructional programs from prekindergarten through grade 12 should enable all students to—	• Recognize and use connections among mathematical ideas • Understand how mathematical ideas interconnect and build on one another to produce a coherent whole • Recognize and apply mathematics in contexts outside of mathematics
Representation Standard Instructional programs from prekindergarten through grade 12 should enable all students to—	• Create and use representations to organize, record, and communicate mathematical ideas • Select, apply, and translate among mathematical representations to solve problems • Use representations to model and interpret physical, social, and mathematical phenomena

Source: Standards are listed with permission of the National Council of Teachers of Mathematics (NCTM). NCTM does not endorse the content or validity of these alignments. Reprinted with permission from *Principles and Standards for School Mathematics,* copyright © 2000 by the National Council of Teachers of Mathematics.

is an important way to add depth of understanding to a newly formed idea.

Throughout this book, this icon will alert you to specific information in *Principles and Standards* relative to the information you are reading. However, these notes and the brief descriptions you have just read should not be a substitute for reading the *Standards* documents. Members of NCTM have access online to the complete *Principles and Standards* document as well as the three previous standards documents. Nonmembers can sign up for 120 days of free access to the *Principles and Standards* at www.nctm.org. The website also contains a number of free applets (referred to as "e-Examples"), which are interactive tools for learning about mathematical concepts. ◆

Curriculum Focal Points: A Quest for Coherence

The goals established by states are sometimes broad and numerous (discussed more thoroughly later in this chapter in the section "Grade-Level Expectations"), often covering many topics in 1 year without clearly indicating how those topics should be connected. Once again, NCTM responded to the needs expressed by teachers of mathematics, state curriculum leaders, and other educators at a variety of agencies to pinpoint mathematical "targets" for each grade level that specify the big ideas for the most significant concepts and skills. NCTM brought together a variety of experts who researched this topic and wrote *The Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence* (2006). This document is organized by grade level and NCTM content strands, emphasizing for each grade three essential areas (Focal Points) as the primary focus of that year's instruction. The topics relating to that focus are organized to show the importance of a *coherent* curriculum rather than a curriculum with a list of isolated topics. The expectation is that those focal points along with integrated process skills and connecting experiences would form the fundamental core content of that grade. The *Curriculum Focal Points* are, in fact, a stimulus for conversations among teachers, administrators, families, and other interested stakeholders about the emphasis, depth, and sequence of key ideas for their child, classroom, school, or state. Not surprisingly, over half the states are already aligning their curriculum with the Focal Points. Besides focusing instruction, the document provides guidance to professionals about ways to refine and streamline the existing curriculum in light of competing priorities.

myeducationlab

Go to the Activities and Application section of Chapter 1 of MyEducationLab. Click on Online Resources and then on the link entitled **"Curriculum Focal Points"** to explore mathematical topics for each grade level.

The Professional Standards for Teaching Mathematics and Mathematics Teaching Today

Although *Principles and Standards* incorporates principles of teaching and assessment, the emphasis is on curriculum. In contrast, *The Professional Standards for Teaching Mathematics* (1991) (available free online to NCTM members) and its companion document, *Mathematics Teaching Today* (2007) (see Appendix B), focus on teaching. Through detailed classroom stories (vignettes) of real teachers, the documents articulate the careful, reflective work that must go into the teaching of mathematics.

Shifts in the Classroom Environment

The introduction to *Mathematics Teaching Today* lists six major shifts in the environment of the mathematics classroom that are necessary to allow students to develop mathematical understanding:

- Communities that offer an equal opportunity to learn to all students
- A balanced focus on conceptual understanding as well as on procedural fluency
- Active student engagement in problem solving, reasoning, communicating, making connections, and using multiple representations
- Well-equipped learning centers in which technology is used to enhance understanding
- Incorporation of multiple assessments that are aligned with instructional goals and practices
- Mathematics authority that lies within the power of sound reasoning and mathematical integrity (NCTM, 2007, p. 7).

The Teaching Standards

Mathematics Teaching Today contains chapters on (1) teaching and learning; (2) observation, supervision, and improvement of mathematics teaching; (3) education and continued professional growth of teachers; (4) working together to achieve the vision; and (5) questions for the reflective practitioner. In the teaching and learning section there are seven mathematics teaching standards:

1. Knowledge of Mathematics and General Pedagogy
2. Knowledge of Student Mathematical Learning
3. Worthwhile Mathematical Tasks

4. Learning Environment
5. Discourse
6. Reflection on Student Learning
7. Reflection on Teaching Practice

Mathematics Teaching Today (and its predecessor) is an excellent resource to help you envision your role as a teacher in creating a classroom that supports the *Principles and Standards.*

Pause and Reflect

The seven teaching standards are located in Appendix B of this book. Take a moment now to look over this one-page listing. Select one or two of the standards that seem especially significant to you. Put a sticky note on the page to remind you to return to these important ideas from time to time as you work through this book.

Influences and Pressures on Mathematics Teaching

NCTM has provided the major leadership and vision for reform in mathematics education. However, no single factor controls the direction of change. National and international comparisons of student performance continue to make headlines, provoke public opinion, and pressure legislatures to call for tougher standards backed by testing. The pressures of testing policies exerted on schools and ultimately on teachers may have an impact on instruction that is different from the vision of the NCTM *Standards.* In addition to these pressures, there is also the strong influence of the textbook or curriculum materials that are provided to teachers, which may not be aligned with state standards.

National and International Studies

Large studies that tell the American public how the nation's children are doing in mathematics receive a lot of attention. They influence political decisions as well as provide useful data for mathematics education researchers. Why do these studies matter? Because international and national assessments provide strong evidence that mathematics teaching *must* change if our students are to be competitive in the global market and able to understand the complex issues they must confront as responsible citizens.

National Assessment of Educational Progress. Since the late 1960s and at regular intervals (2 and 4 years), the United States gathers national data on how students are doing in mathematics (and other content areas) through the National Assessment of Educational Progress (NAEP). These data provide an important tool for policy makers and educators to measure the overall improvement of U.S. students over time. Reported in what is called the "Nation's Report Card," NAEP examines both national and state-level trends. NAEP rates students in grades 4, 8, and 12 using four performance levels: Below Basic, Basic, Proficient, and Advanced (with Proficient and Advanced representing substantial grade-level achievement). The criterion-referenced test is designed to reflect current curriculum. In the most recent assessment in 2007, less than half of all U.S. students in grades 4 and 8 performed at the desirable range between Proficient and Advanced (39 percent in each case) (U.S. Department of Education, 2008). Although the No Child Left Behind legislation expects that all students will be at or above the Proficient level by 2014, NAEP data suggest that goal is probably not attainable. Most troubling, approximately 18 percent of fourth-grade students and 29 percent of eighth-grade students were at the Below Basic level. Despite small gains in the NAEP scores over the last 30 years, U.S. students' performance has remained at discouraging levels of competency (full information can be found at http://nationsreportcard.gov/math_2007).

Trends in International Mathematics and Science Study. In 1995 and 1996, 41 nations participated in the Third International Mathematics and Science Study (TIMSS), the largest study of mathematics and science education ever conducted. Data were gathered in grades 4, 8, and 12 from 500,000 students as well as from teachers. The most widely reported results are that U.S. fourth-grade students are above the average of the TIMSS countries, below the international average at the eighth grade, and significantly below average at the twelfth grade (U.S. Department of Education, 1997a).

In 1999 (38 countries), 2003 (46 countries), and 2007 (63 countries), repeat TIMSS studies were conducted. (The acronym TIMSS now standing for Trends in International Mathematics and Science Study.) The most recent version analyzed (2003) finds that although the rank ordering for fourth grades places the United States above the average, 11 countries (or parts of countries) have significantly higher scores (Singapore, Hong Kong, Japan, Chinese Taipei, Flemish Belgium, Netherlands, Latvia, Lithuania, Russian Federation, England, and Hungary). Only 7 percent of U.S. fourth graders would fall in the Advanced International Benchmark. This is in stark contrast with Singapore at 44 percent, Chinese Taipei at 38 percent, and Japan at 24 percent (Mullis, Martin, Gonzales, & Chrostowski, 2004).

A major finding of the original TIMSS curriculum analysis called the U.S. mathematics curriculum "a mile wide and an inch deep" (Schmidt, McKnight, & Raizen, 1996, p. 62), meaning it was found to be unfocused, pursuing many more topics than other countries while yet involving a great deal of repetition. The U.S. curriculum attempted to do everything and, as a consequence, rarely

provided depth of study, making reteaching all too common (Schmidt et al., 1996). In response, the purpose of the *Curriculum Focal Points* is to assist states and districts in moving away from this "mile wide, inch deep" curriculum to one that is focused and goes into depth at each grade level.

One of the most interesting components of the 1999 study was the inclusion of a video study conducted in eighth-grade classrooms in the United States, Australia, and five of the highest-achieving countries. The results indicate that teaching is a cultural activity, and the differences for countries were often striking despite many similarities. In all countries problems or tasks were frequently used to begin the lesson. However, as a lesson progressed, the way these problems were handled in the United States was in stark contrast to the high-achieving countries. Analysis revealed that although the world is for all purposes unrecognizable from what it was 100 years ago, the U.S. approach to teaching mathematics during the same time frame was essentially unchanged.

Does the following typical U.S. lesson sound at all familiar? The teacher begins with a review of previous materials or homework and then demonstrates a problem at the board. Students practice similar basic problems at their desks, the teacher checks the seatwork, and then assigns further problems for either the remainder of the class session or homework. In more than 99.5 percent of the U.S. lessons the teacher reverts to showing students how to solve the problems. In not one of the 81 videotaped U.S. lessons was any high-level mathematics content observed; in contrast, 30 to 40 percent of lessons in Germany and Japan contained high-level content. As we stated previously, the teachers knew the research team was coming to videotape; nevertheless, 89 percent of the U.S. lessons consisted exclusively of low-level content. In the Czech Republic, Hong Kong, and Japan, lessons incorporated a variety of methods, but they frequently began with a problem-solving approach and continued in that spirit with an emphasis on conceptual understanding and true problem solving (Hiebert et al., 2003). Teaching in the high-achieving countries more closely resembles the recommendations of the NCTM *Standards* than does the teaching in the United States.

State Standards

The term *standards* was popularized by NCTM in 1989. Today it is used by nearly every state in the nation to refer to a grade-by-grade listing of very specific mathematics objectives. These state standards or objectives vary considerably from state to state. Even the grade level at which basic facts for each of the operations are expected to be mastered varies by as much as three grade levels. Although the NCTM *Standards* document lists goals for each of four grade bands, it is not a national curriculum. The United States and Canada are the only industrialized countries in the world without a national curriculum.

Grade-Level Expectations. In 2001 the legislation commonly known as No Child Left Behind (NCLB) was enacted, requiring highly qualified teachers in every classroom, proficiency from all students by 2014, incremental annual achievement based on assessments of adequate yearly progress (AYP), and development by states of content standards that are rigorous and specific. These grade-level learning expectations (GLEs) help guide textbook selection, inform the topics taught and assessed at different grades, and eventually direct what is taught to prospective teachers at universities. But as you might suspect, GLEs vary from state to state—sometimes dramatically (Reys & Lappan, 2007). For example, just in total numbers alone, at the fourth-grade level Florida has 89 GLEs in mathematics and North Carolina has 26. Textbook publishers try to cover as many states' requirements as possible, particularly populous states, in order to maximize sales of textbooks. However, this burdens teachers who must sort through many topics and corresponding lessons in a given book to eliminate some materials while sometimes needing to supplement the text with other resources to cover missing topics. Researchers also point out that textbooks' "limited overlap" and "large number of unique learning expectations" result in shallow treatment of many topics (Reys, Chval, Dingman, McNaught, Regis, & Togashi, 2007, p. 11). As more states consider such research in combination with the NCTM *Curriculum Focal Points*, we hope that collaboration may yield consensus and a narrowing of emphasis or focus will occur.

Assessments. Associated with every set of state standards is some form of testing program. Publicly reported test scores place pressure on superintendents, then on principals, and ultimately on teachers, who feel enormous pressure to raise test scores at all costs (Schmidt et al., 1996). For a teacher who has little or no experience with the spirit of the *Standards*, it is very difficult to adopt the student-centered approach to mathematics when preoccupied with preparing for high-stakes tests. Unfortunately for children, the resulting drill, review, and practice tests produce mathematics experiences with little or no high-level thinking, problem solving, or reasoning.

Are state standards incompatible with the *Standards*? Good mathematics teaching is about helping children understand concepts and become confident in their abilities to do mathematics and solve problems. There are many wonderful examples of teaching in the spirit of the NCTM standards. Children in these classrooms achieve quite well, even on the most traditional of standardized tests.

Curriculum

In most classrooms, the textbook is the single most influential factor in determining the what, when, and how of actual teaching. What is becoming increasingly complicated is how teachers and school systems attempt to align

the textbook or other curriculum materials with the mandated state pre-standards. Though possibly an oversimplification, mathematics curriculum materials that are used in pre-K–8 classrooms can be categorized as either traditional or standards-based—meaning reflecting the spirit of the NCTM *Standards*.

myeducationlab

Go to the Activities and Application section of Chapter 1 of MyEducationLab. Click on Online Resources and then on the link entitled "**Standards-Based Curricula**" for a list of standards-based curricula and their developers, publishers, and Internet contacts.

Traditional Curricula. The term "traditional textbook" is used here to describe books that are developed by major publishing companies based on market research. Though traditional textbooks vary in some ways among one another, there are several characteristics that tend to be true for all of them. First, traditional textbooks reflect publishers' efforts to cover the topics in every state's curriculum documents. Since states vary widely in the topics they include at a particular grade level, this approach of including everything results in a very large textbook with many, many topics. Second, because there are so many topics, most of them are covered in a one-day lesson, which may be inadequate in developing a deep understanding. Third, traditional texts incorporate the implied instructional model of the teacher demonstrating and explaining how to do the mathematics and students then practicing those procedures. Fourth, and perhaps most challenging in terms of the international research previously discussed, is the traditional emphasis on mathematical procedures at the expense of conceptual understanding. For example, in a unit on fractions, a traditional text is likely to focus on showing students how to do the computation rather than focus on when that computation might be needed or how that topic is related to other mathematics strands.

Textbooks greatly influence teaching practice. A teacher using a traditional textbook is more likely to cover many topics, spend one day on each topic, use a teacher-directed instructional approach, and focus on procedures. If a teacher wants to devote more time to a concept, teach it more deeply, or focus on conceptual understanding, for example, he or she may need to adapt and extend the lessons in the textbook.

Standards-Based Curricula. In contrast to traditional textbooks, standards-based textbooks are not based on market research but on research related to how students learn mathematics and how concepts should develop over time. Therefore, they tend to cover fewer topics, spend more time on each concept, and make connections among concepts. Many of the standards-based programs are designed for students to learn through inquiry-oriented approaches—not through teacher explanation. Finally, all of the standards-based programs have a strong emphasis on conceptual understanding (not just procedures) and on solving problems.

At present, there are three elementary and four middle school programs commonly recognized as standards-based curricula. A hallmark of these standards-based or alternative programs is student engagement. Children are challenged to make sense of new mathematical ideas through explorations and projects, often in real contexts. Written and oral communication is strongly encouraged.

Data concerning the effectiveness of standards-based curricula as measured by traditional testing programs continue to be gathered. It is safe to say that students in standards-based programs perform much better on problem-solving measures and at least as well on traditional skills as students in traditional programs (Bell, 1998; Boaler, 1998; Fuson, Carroll, & Drueck, 2000; Hiebert, 2003; Reys, Robinson, Sconiers, & Mark, 1999; Riordin & Noyce, 2001; Stein, Grover, & Henningsen, 1996; Stein & Lane, 1996; Wood & Sellers, 1996, 1997).

Because textbooks are so central in current teaching, use of a standards-based textbook strongly influences what teachers do. Interesting and meaningful tasks are easily accessible, so the teacher is much more likely to have math lessons that link important mathematics concepts to contexts that engage students. The teacher is more likely to spend more time on concepts rather than an exclusive focus on procedures, because the student investigations are conceptually oriented. Writing, speaking, working in groups, and problem solving are more likely to be commonplace components. Comparing any of these activities to procedures associated with a corresponding traditional textbook would be an effective way to understand what reform or standards-based mathematics is all about.

In Chapters 9, 14, 18, and 19 of Section 2 you will find features describing activities from two standards-based programs: *Investigations in Number, Data, and Space* (Grades K–5) or *Connected Mathematics* (Grades 5–8). These features are included to offer you some insight into these nontraditional programs as well as to offer good ideas for instruction.

A Changing World Economy

The Glenn Commission Report, headed by former astronaut and senator John Glenn, states, "60% of all new jobs in the early 21st century will require skills that are possessed by only 20% of the current workforce" (U.S. Department of Education, 2000, p. 11). The report found that schools are not producing "graduates with the kinds of skills our economy needs to remain on the competitive cutting edge" (p. 12). These skills are often the mathematical skills that build the infrastructure of our nation.

In his book *The World Is Flat* (2007), Thomas Friedman discusses the need for people to have skills that are lasting and will survive the ever-changing landscape of available jobs. These are what he calls "the untouchables"—the individuals who will make it through all economic revolutions. He suggests that if people can fit into several of the broad categories

he defines then they will not be challenged by a shifting job market. One of these safety-ensuring categories in his analysis is "math lovers." Friedman points out that in a world that is digitized and surrounded by algorithms, the math lover will always have opportunities and options. Now it becomes the job of the teacher to develop this passion in students. As Lynn Arthur Steen, a well-known mathematician and educator, states, "As information becomes ever more quantitative and as society relies increasingly on computers and the data they produce, an innumerate citizen today is as vulnerable as the illiterate peasant of Gutenberg's time" (1997, p. xv).

The changing world influences what should be taught in pre-K–8 mathematics classrooms. As we prepare elementary students for jobs that possibly do not currently exist, we do know that there are few jobs for people where they just do simple computation. We can predict that there will be work that requires interpreting complex data, designing algorithms to make predictions, and using the ability to approach new problems in a variety of ways.

An Invitation to Learn and Grow

The mathematics education described in the NCTM *Standards* may not be the same as the mathematics and the mathematics teaching you experienced in grades K through 8. Along the way, you may have had some excellent teachers who really did reflect the current reform spirit. Examples of good standards-based curriculum have been around since the early 1990s, and you may have benefited from one of them. But for the most part, the goals of the reform movement at the end of its second decade have yet to be realized in the large majority of school districts in North America.

As a practicing or prospective teacher facing the challenge of the *Standards*, this book may require you to confront some of your personal beliefs—about what it means to *do mathematics*, how one goes about *learning mathematics*, how to *teach mathematics through problem solving*, and what it means to *assess mathematics* integrated with instruction.

As part of this personal assessment, you should understand that mathematics is seen as the subject that people love to hate. At parties or even at parent–teacher conferences, other adults will respond to the fact that you are a teacher of mathematics with comments such as "I could never do math," or "I can't even balance my checking account." Instead of just dismissing these disclosures, they are not to be taken lightly. Would people confide that they don't read and hadn't read a book in years? That is not likely. Families' and teachers' attitudes toward mathematics may enhance or detract from children's ability to do math. It is important for you and for students' families to know that math ability is not inherited—anyone can learn mathematics. Moreover, learning mathematics is an essential life skill. You need to find ways of countering these statements, especially if they are stated in the presence of children, pointing out the importance of the topic and the fact that all people have the capacity to learn it. Only in that way can the long-standing pattern that passes this apprehension from family member to child (or in rare cases teacher to child) be broken. There is much joy to be had in solving mathematical problems, and you need to nurture that passion in children.

Children and adults alike need to think of themselves as mathematicians, in the same way as they think of themselves as readers. As all people interact with our increasingly mathematical and technological world, they need to construct, modify, or integrate new information in many forms. Solving novel problems and approaching circumstances with a mathematical perspective should come as naturally as reading new materials to comprehend facts, insights, or news. Thinking and talking about mathematics instead of focusing on the "one right answer" is a strategy that will serve us well in becoming a society where all citizens are confident that they can do math.

Becoming a Teacher of Mathematics

This book and this course of study are critical to your professional teaching career. The mathematics education course you are taking now will be the foundation for much of the mathematics instruction you do in your classroom for the next decade. The authors of this book take that seriously, as we know you do. Therefore, this section lists and describes the characteristics, habits of thought, skills, and dispositions you will need to succeed as a teacher of mathematics.

myeducationlab

Go to the Activities and Application section of Chapter 1 of MyEducationLab. Click on Videos and watch the video entitled "**John Van de Walle on Approach to Teaching**" to see him speak about his approach to teaching mathematics.

Knowledge of Mathematics. You will need to have a profound, flexible, and adaptive knowledge of mathematics content (Ma, 1999). This statement is not intended to scare you if you feel that mathematics is not your strong suit, but it is meant to help you prepare for a serious semester of learning about mathematics and how to teach it. The "school effects" for mathematics are great, meaning that unlike other subject areas, where children have frequent interactions with their family or others outside of school on topics such as reading books, exploring nature, or discussing current events, in the area of mathematics what we do in school is "it" for many children. This adds to the earnestness of your responsibility, because a student's learning for the year in mathematics will likely come only from you. If you are not sure of a fractional concept or other aspect of the mathematics curriculum, now is the time to make changes in your depth of understanding to best prepare for

your role as an instructional leader. This book and your professor will help you in that process.

Persistence. You need the ability to stave off frustration and demonstrate persistence. This is the very skill that your students must have to conduct mathematical investigations. As you move through this book and work the problems yourself, you will learn methods and strategies that help you anticipate the barriers to student learning and identify strategies to get past these stumbling blocks. It is likely that what works for you as a learner will work for your students. As you experience the material in this book, if you ponder, struggle, talk about your thinking, and reflect on how it all fits or doesn't fit, then you enhance your repertoire as a teacher. Remember you need to demonstrate these characteristics so your students can model them.

Positive Attitude. Arm yourself with a positive attitude toward the subject of mathematics. Research shows that teachers with positive attitudes teach math in more successful ways that result in their students liking math more (Karp, 1991). If in your heart you say, "I never liked math," that will be evident in your instruction. The good news is that research shows that attitudes toward mathematics are relatively easy to change (Tobias, 1995) and that the changes are long-lasting. Through expanding your knowledge of the subject and trying new ways to approach problems, you can learn to enjoy mathematical activities. Not only can you acquire a positive attitude toward mathematics, it is essential that you do.

Readiness for Change. Demonstrate a readiness for change, even for change so radical that it may cause disequilibrium. You may find that what is familiar will become unfamiliar and, conversely, what is unfamiliar will become familiar. For example, you may have always referred to "reducing fractions" as the process of changing $\frac{2}{4}$ to $\frac{1}{2}$, but is "reducing" what is going on conceptually? Are reduced fractions getting smaller? Such terminology can lead to mistaken connections that children will naturally make ("Did the reduced fraction go on a diet?"). A careful look will point out that "reducing" is not a good term to use when focusing on conceptual knowledge. Even though you have used this familiar expression for years, it is inappropriate, because it does not explain what is really happening. We will discuss innovative and conceptually sound methods for teaching fractions in Chapter 15.

On the other hand what is unfamiliar will become more comfortable. It may feel uncomfortable for you to be asking students, "Did anyone solve it differently?" if you are worried that you won't understand their explanations. Yet bravely using this strategy will lead you to understand the concept better yourself as you ask students to re-explain how they solved a problem so that you can understand their thinking.

Another potentially difficult change is toward a focus on concepts. What happens in a procedure-focused classroom when a student doesn't understand division of fractions? A teacher who only has procedural knowledge is often left with just one approach: repeating, louder and slower. "Just change the division sign to multiplication, flip over the second fraction, and multiply." We know this approach doesn't work well, so let's think about another. Consider $3\frac{1}{2} \div \frac{1}{2} =$ _____. In a conceptual approach, you might relate to a whole number problem such as $25 \div 5 =$ _____. A corresponding story problem might be, "How many orders of 5 pizzas are there in a group of 25 pizzas?" Returning to the fraction problem, ask students to put words around the division problem, such as "You plan to serve each guest $\frac{1}{2}$ a pizza. If you have $3\frac{1}{2}$ pizzas, how many guests can you serve?" Yes, there are seven halves in $3\frac{1}{2}$ and therefore 7 guests you can serve. Are you surprised that you can do this problem mentally?

To respond to students' challenges, uncertainties, and frustrations you need to unlearn and relearn mathematical concepts, developing comprehensive understanding and substantial representations along the way. Supporting your knowledge on solid, well-supported terrain is your best hope of making a lasting difference—so be ready for change. What you already understand will provide you with many "Aha" moments as you read this book and connect new information to the mathematics knowledge currently stored in your memory.

Reflective Disposition. Make time to be self-conscious and reflective. As Steve Leinwand, the former director of mathematics education in Connecticut, wrote, "If you don't feel inadequate, you're probably not doing the job" (2007, p. 583). No matter if you are a preservice teacher or an experienced teacher, there is more to learn about the content and methodology of teaching mathematics. The ability to examine oneself for areas that need improvement or to reflect on successes and challenges is critical for growth and development. The best teachers are always trying to improve their practice through the latest article, the newest book, the most recent conference, or by signing up for the next series of professional development opportunities. These teachers don't say, "Oh, that's what I am already doing"; instead, they identify and celebrate one small tidbit that adds to their repertoire. The best teachers never finish learning all that they need to know, they never exhaust the number of new connections that they make, and, as a result, they never see teaching as stale or stagnant. An ancient Chinese proverb states, "The best time to plant a tree is twenty years ago; the second best time is today." So, as John Van de Walle said with every new edition, "Enjoy the journey!"

Reflections on Chapter 1

Writing to Learn

At the end of each chapter of this book, you will find a series of questions under this same heading. The questions are designed to help you reflect on the most important ideas of the chapter. Writing (or talking aloud with a peer) is an excellent way to explore new ideas and incorporate them into your own knowledge base. The writing (or discussion) will help make the ideas your own.

1. What are the five content strands (standards) defined by *Principles and Standards*? How are they emphasized differently in different grade bands?
2. What is meant by a *process* as referred to in the *Principles and Standards* process standards? Give a brief description of each of the five process standards.
3. Among the ideas in *Mathematics Teaching Today* are six shifts in the classroom environment. Examine these six shifts, and describe in a few sentences what aspects of each shift seem most significant to you.
4. Describe two results derived from NAEP data. What are the implications?
5. Describe two results derived from TIMSS data. What are the implications?

For Discussion and Exploration

1. In recent years, the outcry for "basics" was again being heard from a variety of sources. The debate between reform and the basics is both important and interesting. For an engaging discussion of the reform movement in light of the "back to basics" outcry, read the three free online articles from the February 1999 edition of the *Phi Delta Kappan* at www.pdkintl.org/kappan/khome/karticle.htm. Where do you stand on the issue of reform versus the basics?
2. Examine a traditional textbook at any grade level of your choice. If possible, use a teacher's edition. Page through any chapter and look for signs of the five process standards. To what extent are children who are being taught from this book likely to be doing and learning mathematics in ways described by those processes? What would you have to do to supplement the general approach of this text?
3. Examine a unit from any one of the standards-based curriculum programs and see how it reflects the NCTM vision of reform, especially the five process standards. How do these curriculum programs differ from traditional textbook programs? Do you need to supplement this text?

Resources for Chapter 1

Recommended Readings

Articles

Hoffman, L., & Brahier, D. (2008). Improving the planning and teaching of mathematics by reflecting on research. *Mathematics Teaching in the Middle School*, *13*(7), 412–417.
This article addresses how a teacher's philosophy and beliefs influence his or her mathematics instruction. Using TIMSS and NAEP studies as a foundation, the authors talk about posing higher-level problems, asking thought-provoking questions, facing students' frustration, and using mistakes to enhance understanding of concepts. They pose a set of reflective questions that are good for self-assessment or discussion with peers.

Books

Ferrini-Mundy, J. (2000). The standards movement in mathematics education: Reflections and hopes. In M. J. Burke (Ed.), *Learning mathematics for a new century* (pp. 37–50). Reston, VA: NCTM.
In this chapter, written before Standards *was released, the author shares her unique and very well-informed view of this important publication, how it came to be, the impact of the earlier document, the political climate in which* Standards *was released, and the intentions that NCTM had for the document. This article will provide an understanding of* Standards *that is impossible to get from the document itself.*

Hiebert, J. (2003). What research says about the NCTM standards. In J. Kilpatrick, W. G. Martin, & D. Schifter (Eds.), *A research companion to Principles and Standards for School Mathematics* (pp. 5–23). Reston, VA: NCTM.
This chapter provides one of the best perspectives on what we have learned since Standards *was released. It also offers some perspective on typical U.S. classrooms and offers contrasts between traditional mathematics programs and those called "standards based."*

National Research Council. (2001). *Adding it up: Helping children learn mathematics.* J. Kilpatrick, J. Swafford, & B. Findell (Eds.). Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academy Press.
This book is the effort of a select committee representing mathematics educators, mathematicians, school administrators, and industry. A hallmark of this book is the formulation of five strands of "mathematical proficiency": conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and

productive disposition. Educators and policy makers will cite this book for many years to come.

Standards-Based Curricula

Elementary Programs

UCSMP Elementary: Everyday Mathematics (K–6)
Investigations of Number, Data, and Space (K–5) (samples included throughout the book)
Math Trailblazers: A Mathematical Journey Using Science and Language Arts (K–5)

Middle School Programs

Connected Mathematics (CMP) (6–8) (samples included throughout the book)
Mathematics in Context (MIC) (5–8)
MathScape (6–8)
Middle Grades Math Thematics (STEM) (6–8)
Middle School Mathematics Through Applications Project (MMAP) (6–8)

Online Resources

Illuminations
www.illuminations.nctm.org
A companion website to NCTM sponsored by NCTM and Marcopolo. Provides lessons, interactive applets, and links to websites for learning and teaching mathematics.

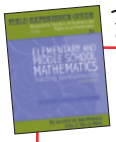
Key Issues in Math
www.mathforum.org/social/index.html
Part of the Math Forum at Drexel University, this page lists numerous questions concerning issues in mathematics education with answers supplied by experts in short articles or excerpts.

NAEP (National Assessment of Educational Progress, "The Nation's Report Card")
http://nces.ed.gov/nationsreportcard/mathematics
Past and current data and reports related to NAEP assessments.

National Council of Teachers of Mathematics
www.nctm.org
Here you can find all about NCTM, its belief statements, and positions on important topics. Also find an overview of *Principles and Standards* and free access to interactive applets (see Standards—Electronic), membership and conference information, publications catalog, links to related sites, and much more. Members have access to even more information.

State Mathematics Standards Database
http://mathcurriculumcenter.org/states.php
This site from The Center for the Study of Mathematics Curriculum (CSMC) has the complete set of hotlinks to current state-level K–12 mathematics curriculum standards. In some cases states provide multiple documents, including their standards for assessment or other important information for teachers of mathematics.

TIMSS (Trends in International Mathematics and Science Study)
http://nces.ed.gov/timss
Access articles and data from TIMSS.



Field Experience Guide Connections

The *Field Experience Guide: Resources for Teachers of Elementary and Middle School Mathematics* (FEG) is a workbook designed to respond to both the variety of teacher preparation programs and the NCTE recommendation that students have the opportunity to engage in diverse activities. At the end of each chapter, you will find a brief note that connects chapter content to activities and experiences within the guide. Many of the field experiences focus on aligning practice with the standards. For example, see the observation protocol for shifts in the classroom environment (FEG 1.2), a teacher interview based on the teaching standards (FEG 1.3), and observation protocol for the process standards (FEG 4.1). Developing a reflective disposition is the purpose of FEG 3.7, 4.8, 5.5, and 6.4. These opportunities for reflection focus on your students' learning and your own professional growth.