Why Whole Class Discussions?

As students are asked to communicate about the mathematics they are studying—to justify their reasoning to classmates or formulate a question about something that is puzzling—they gain insights into their thinking. In order to communicate their thinking to others, students naturally reflect on their learning and organize and consolidate their thinking about the mathematics.

(NCTM, Principals and Standards)

Teachers interested in helping students learn math more effectively have many questions when approached with the concept of using whole class discussions to facilitate understanding mathematics: What do students gain from whole class discussions? How can you support students in learning math through whole class discussions? What do you need to do in order to effectively use whole class discussions in your classroom? By asking these questions, teachers are able to explore what research already shows: that students learn math through communicating. When they discuss their ideas, listen to different perspectives, and engage in sense-making, students deepen their understanding of mathematics. Mathematics is more than following a series of steps and memorizing formulas. Mathematics is a “dynamic discipline focused on solving problems by thinking creatively, finding patterns, and reasoning.
logically” (Bray, 2011). Teaching through a problem-solving approach that focuses on student understanding can help students learn math. Whole class discussions to support student learning are essential in NCTM standards-based programs (Kazemi & Stipek, 2001; Stein, Engle, Smith, Hughes, 2008). My goal in this book is to share effective whole class discussion strategies that will help you support your students as they learn mathematics with conceptual understanding. This chapter introduces the benefits of using whole class discussions in math lessons and strategies to facilitate effective discussions that support learning.

Common Core and NCTM Standards

The Common Core State standards for Mathematics and the National Council of Teachers of Mathematics (NCTM) standards highlight the importance of communication in helping students develop deep understanding of mathematical ideas and their ability to proficiently solve problems. To meet these standards, students must be actively engaged in the process of learning mathematics. The standards cover not only the mathematical concepts that students should learn, but also outline an approach for teaching mathematics. For example, the Common Core standards include eight mathematical practices detailing how students should learn mathematics. These eight practices also align with the NCTM process standards. In both the Common Core State and NCTM standards, communication is an essential part of how students should learn math, as shown in this Common Core standard of mathematical practices:

3. Construct viable arguments and critique the reasoning of others.
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases and can recognize counter-examples. They justify their conclusions and communicate them to others and respond to the argument of others. . . . (CCSSO & NGA, 2010)

According to the Common Core State standards, students must be able to explain and justify their thinking. In addition, students should be able to explain why a mathematical statement is true or how a mathematical rule works.

NCTM process standards integrate problem solving, reasoning and proof, communication, and representations. The NCTM (2000) Communication standard states that students should be able to:
• organize and consolidate their mathematical thinking through communication
• communicate their mathematical thinking coherently and clearly to peers, teachers and others
• analyze and evaluate the mathematical thinking and strategies of others
• use the language of mathematics to express ideas precisely

By focusing on how students can communicate mathematical concepts, both the Common Core State standards and the NCTM standards naturally support a whole class discussion approach to teaching math. It will allow students to develop more efficient mathematical thinking and to build on concepts they have discovered through effective, guided communication.

**Whole Class Discussion and Student Achievement**

Whole class discussions take time. Many teachers wonder if the time spent on whole class discussion is justified, especially when so much content must be covered in a short period of time. However, when students develop conceptual understanding of mathematical ideas, they retain what they learn and develop greater skills in math. If students forget how to solve a problem, they can figure it out if they understand the concepts behind the problem. On the other hand, when students simply memorize problem-solving procedures without understanding how or why they work, they are more likely to have difficulty remembering how to do problems (Kilpatrick, Swafford & Bradford, 2001). For example, students who memorize that $9 + 9 = 18$ without understanding what it means are less likely to apply that type of problem solving to additional problems.

Helping students understand mathematical concepts (known as having number sense) allows students to make mathematical connections and successfully solve problems using efficient strategies.

Whole class discussions can help students develop their number sense. Researchers have found that focusing on different reasoning strategies leads to higher mathematical insights (Stein, Engle, Smith & Hughes, 2008; Leinhardt & Steele, 2005). Furthermore, students who learn math through an effective standards-based approach (such as the Common Core or NCTM standards) do well in math achievement tests and outperform students who are taught using traditional approaches in problem-solving tasks (Schoenfeld, 2002).
Schoenfeld (2002) points out that these approaches to teaching also narrow the achievement gap among English language learners. When these students have the opportunity to speak in their first language for a few minutes, transition to English, and connect the words to visual representations, they make vocabulary connections (Wiest, 2008). ELLs are faced with learning a second language at the same time that they are learning the content. Therefore, teachers need to think about students’ language goals as well as the mathematical goals (Bresser, Melanese & Sphar, 2009). Teachers can do this by identifying vocabulary that they are unfamiliar with and find ways to build discussion based on what students know. This strategy is useful for helping all students communicate around difficult math vocabulary until they have firmly grasped the new concept. Once they conceptually understand the lesson, integrating the math vocabulary will give students greater math communication abilities and help them meet state standards.

**Communication and Mathematical Learning**

The National Research Council has identified five strands for mathematical proficiency (Kilpatrick, Swafford, & Findell, 2001):*

1. *conceptual understanding*—comprehension of mathematical concepts, operations, and relations
2. *procedural fluency*—skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
3. *strategic competence*—ability to formulate, represent, and solve mathematical problems
4. *adaptive reasoning*—capacity for logical thought, reflection, explanation, and justification
5. *productive disposition*—habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy.

All five strands require communication. Consider the requirements for reflection and explanation in the fourth strand, adaptive reasoning. When students share their reasoning and listen to different perspectives, they engage in *reflective* thought. Reflective thought involves carefully analyzing what is being said, making judgments, and understanding how

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that new information fits with prior knowledge. On the other hand, if new information does not fit, reflective thought requires that existing ideas be adjusted with the new ones (Fosnot, 2005). Typically, new learning and deeper understanding result from this reflective thinking process. By sharing, explaining, and justifying ideas, students develop skills in reasoning and engage in sense-making.

“Math Talk” with Partners, Small Groups, and Whole Class

There are many different types of communication that can take place during a math lesson, such as talk with partners, small groups, and the whole class. Each type of “math talk” serves a different purpose and contributes to the development of new mathematical insights. For example, students can brainstorm or clarify ideas with partners, small groups, or the teacher, and discuss ideas as a group to explore a topic in more depth. Although this book focuses on whole class discussions, it also explains the importance of working with partners or small groups to allow students to individually share their thinking, get specific feedback on their ideas, and reflect on other ideas. The drawback of partner or small group talk is that students are limited to the perspective of the partner or small group. Students may not be able to solve problems themselves and clear up any misconceptions during these conversations without the additional support of a whole class discussion.

Case Study: Problem Solving with Partners

A first-grade class is asked to share their strategies with a partner for adding $9 + 9$. Melissa uses counters to solve the problem. She takes 9 blocks out of a basket and places them on her desk. Then she takes out another 9 blocks, places them on the table, and starts counting by pointing to the blocks. However, since she is not physically moving the blocks, she loses track of which blocks she has counted and gets the answer 15.

Her partner, Martin, also takes out 9 blocks. Then he takes another 9 blocks and adds them to the first 9 blocks to make one large pile. He carefully moves each block to the other side of the table and counts them one at a time. After counting all the blocks, he concludes that the answer is 18. Noticing that both students have

Continues on next page
Why Whole Class Discussions?

The richness of a whole class discussion is dependent on the quality of small group and partner conversations that take place first. The small group discussion and partner talks generate a rich background of ideas for a whole class discussion. These smaller discussions activate prior knowledge and engage students in thinking about the lesson. Then, during the whole class discussion, students are exposed to multiple perspectives that stretch their thinking. The teacher’s role is to pose questions and scaffold the discussion by making decisions about how to sequence or facilitate the discussion so that there is a logical progression of ideas that everyone can understand. Eventually, the teacher wants all her students, through reflecting on the various ideas presented, to develop more efficient strategies and deeper understanding of math concepts.

Case Study, continued

different answers, the teacher asks Melissa and Martin to figure out which answer is correct. They start by placing all the blocks in the basket and then counting out two piles of 9 blocks. Martin gathers the blocks together and moves the pieces while Melissa counts. They both conclude that the answer should be 18. Even though Melissa changes her mind that the answer should be 18, she does not realize that she needs to explicitly think about more efficient ways to keep track while she counts.

At another table, Zack and Reilly use completely different strategies. Zack takes out only 9 blocks. He explains to Reilly, “I started with the number 9 and counted up to get the answer (18).” Reilly tells Zack that she figured out her answer in a different way. She tells Zack that 9 + 1 make 10. Therefore, she re-conceptualizes the problem as 10 + 8 = 18.

ANALYSIS: The partner discussion allowed students to discuss and articulate their thinking to their partners, but it would have been beneficial to hear how other students solved the problem. Martin and Melissa could have demonstrated how they counted 18 blocks; Zack could have shown why he counted 9 blocks, which was more efficient than Martin’s way. Reilly could explain how her approach was similar to Zack’s, because she re-conceptualized the problem. The rest of the students could listen, ask questions, explain thinking, and make their own connections to deepen their understanding of addition strategies and develop their number sense.
SELF-REFLECTION QUESTIONS

1. What kinds of math talk currently takes place in your classroom?
2. When do you use partner talk, small group talk, and whole class discussions?
3. What is your purpose in using these different kinds of talk?

A Peek Inside the Classroom

In Video Clip 1.1, Student Interview: Student Perspectives on “Math Talk” and Math Learning, students in a fifth-grade and sixth-grade class explain how “math talk” contributes to mathematical learning. When they discuss their ideas with a partner, it helps them reflect on their own thinking and self-correct any mistakes. Furthermore, it helps them get started on problem solving by clarifying any information they don’t understand. Listening to multiple perspectives during the whole group discussion stretches their thinking. One student said that when she was working with her partner, they were following the wrong path to solve the problem. However, when they listened to the whole group, they were able to understand what they were doing wrong and self-correct.

Their perspective is consistent with existing research. Talk helps students to understand mathematics more deeply and with greater clarity (Chapin, O’Connor & Anderson, 2009).

Video Clip 1.2, Fifth & Sixth Grade Teacher Perspectives on Class Discussions, contains an interview with two teachers who team-teach the students in Video Clip 1.1. Ms. Bertolone-Smith and Ms. Malone combine their fifth- and sixth-grade classes to teach math. Ms. Bertolone-Smith believes that whole class discussion is the most important part of the math lesson because students listen to multiple perspectives, clarify their thinking, and even change their answers. Ms. Bertolone-Smith and Ms. Malone also find that students are motivated to learn when they are mentally engaged. When students have
had time to think about a problem individually and in small groups, they are ready and eager to share their thinking during the discussion.

*Ms. Bertolone-Smith:* They learn so much from watching their peers discuss and analyze the problem. If Marlene or I were to get up there and say, “this is how it is,” you would see them all fade into the background. But when they are listening to their peers describe mathematics that they have engaged in, they are just interested. . . . They are wondering, “What does another person see in the problem that I didn’t?” That is a really rich discussion versus, “I am going to come up and show you the right answer.”

*Ms. Malone:* That piece of “can I listen to their reasoning, can I let go of my reasoning because my reasoning isn’t clear, am I willing to hear someone else and say, Oh I see it!,” This is where some of the most profound changes happen. . . . You notice that we never say what the answer is until it is basically so obvious. . . . Eventually at the end we allow kids to change their answers and I think that is very powerful.

Students need to listen with an open mind and be willing to change their answer if it does not make sense. The whole class discussion opens students to different possibilities that they may not have thought about individually or with small groups. The social interaction leads students to think, reflect, and refine their thought processes (Chapin, O’Conner & Anderson, 2009).

The teachers state that they create “depth and richness in discussion” when they begin with small group and partner talk, and then build a whole class discussion. Partner talk and small group discussion involves fertilizing a topic. When students think about a problem individually and talk with partners and small groups they are throwing themselves at the problem, and making their ideas explicit without needing to be right. This step is critical in building productive whole class discussions. Teachers have found that when students are not given time to think about a problem, the whole class discussion does not work well. Students may not participate in the discussion or give meaningful responses and the teacher is left struggling to explain the ideas.
Characteristics of Effective Whole Class Discussions

Students develop an understanding of math when they talk about math, listen to others, and observe a variety of representations (Wiest, 2008). Whole class discussions must lead to the development of a more sophisticated understanding of mathematics and build on student thinking by addressing different strategies and errors that students make. Classroom discussion time must be used effectively and efficiently. Table 1.1 distinguishes characteristics of two discussions, one that supports conceptual understanding and one that does not.

The biggest difference between these types of discussions is the purpose of discussion. The main purpose of a whole class discussion is to help students develop a conceptual understanding of mathematics and more efficient strategies through mathematical reasoning and sense-making. Students in a whole class discussion that supports mathematical learning are active participants. The discussion builds on student responses (Stein et al. 2008). Let’s look at a few characteristics of effective class discussions that support learning.

_Students Develop “Shared” Understanding of a Problem._ Students in the whole class discussion develop a shared understanding of a problem or

<table>
<thead>
<tr>
<th>TABLE 1.1</th>
<th>Characteristics of Effective and Ineffective Whole Class Discussions</th>
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<tbody>
<tr>
<td><strong>Effective Discussion</strong></td>
<td><strong>Ineffective Discussion</strong></td>
</tr>
<tr>
<td>Discussion is open ended with many opportunities for students to present multiple perspectives.</td>
<td>Discussion is closed, with only one right approach for solving the problem.</td>
</tr>
<tr>
<td>Students engage in sense-making and explore a problem or issue.</td>
<td>Students passively listen to the teacher.</td>
</tr>
<tr>
<td>The purpose of the discussion is to explore and understand a mathematical problem or issue.</td>
<td>Teacher shows students the steps on how to solve problems. Students copy and use the steps to solve problems without understanding why.</td>
</tr>
<tr>
<td>Teacher acts as a facilitator by scaffolding the discussion in order to lead students to make connections. Students explore how to solve problems and understand why the approach works.</td>
<td>Teacher explains steps to problem solving as students listen passively or give the correct answers without further discussion.</td>
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</table>
issue when they offer their perspectives and ask for clarification if they don’t understand something that is being discussed (Cobb, Wood & Yackel, 1993; Franke, Webb & Chan, 2009). This shared understanding allows students to explore a problem and progressively deepen their understanding through discussion. They develop new mathematical insights by discussing errors and misconceptions, and exploring why an idea does or does not make sense (Bray, 2011; Kazemi & Stipek, 2001). Whole class discussions are also appropriate when the teacher is introducing a new concept or is challenging students to think deeper about something they already know.

**Teachers Provide Students with “Guided Intervention.”** Whole class discussions provide teachers with the opportunity for “guided intervention” (Gravemeijer & van Galen, 2003). Students don’t necessarily discover efficient strategies or deeper mathematical understandings on their own. Therefore, class time can be efficiently used when the teacher uses questions to guide students to think about structured concepts and problems in addition to the questions students raise themselves.

**Students Evaluate and Analyze Their Thinking and Their Peers’ Thinking.** The NCTM standards (2000) state that discussion should involve analyzing and evaluating mathematical thinking of others. Students must think about how to approach the problem as well as understand how other students approach it. Whole class discussions must engage students in critical thinking (Chapin, O’Conner & Anderson, 2009; Smith & Stein, 2011). If students think that the teacher is the only one with the correct answer, they are not likely to be mentally engaged.

**Planning a Whole Class Discussion**

Orchestrating an effective whole class discussion involves planning the lesson, identifying a topic and a problem for discussion, allowing students to share reasoning, and using guided questions to facilitate the discussion. The whole class discussion must be situated in larger mathematical goals. The discussion has three phases: (1) making thinking explicit, (2) analyzing each other’s solutions, and (3) developing new mathematical insights. Table 1.2 briefly describes the process of planning and facilitating an effective whole class discussion. The following chapters discuss each phase in more detail and are supported by Video Clips that demonstrate the process in action. Teacher and student interviews illustrate the process of planning a lesson and impact on student learning.
**TABLE 1.2**

<table>
<thead>
<tr>
<th>Planning Prior to Discussion</th>
<th>Identify long-term and short-term goals, consider curricular tasks, and anticipate student reasoning.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Planning During the Lesson</td>
<td>Assess student reasoning/errors/misconceptions. Identify topic for discussion/problem to discuss and think about how to sequence discussion.</td>
</tr>
<tr>
<td>Whole Class Discussion: Facilitating Mathematical Connections</td>
<td>Pose question/issue to start discussion.</td>
</tr>
<tr>
<td></td>
<td>Phase 1: Making thinking explicit</td>
</tr>
<tr>
<td></td>
<td>Phase 2: Analyzing solutions</td>
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<tr>
<td></td>
<td>Phase 3: Developing new mathematical insights. The teacher facilitates the discussion through questioning.</td>
</tr>
</tbody>
</table>

**Planning Prior to Discussion**

A whole class discussion focuses on one concept or goal; however, it is also part of a larger conversation that takes place over time. When students see connections within a lesson and across lessons, they develop deeper mathematical connections. Therefore, planning begins with setting a clear goal and purpose that fits in with the larger purpose. (Chapter 4 discusses lesson planning and sequencing in more detail.) During the planning process the teacher identifies the concepts and skills that students should develop and selects tasks for students to engage in. In addition, the teacher anticipates student reasoning, errors, and misconceptions that can emerge during the lesson.

**A Peek Inside the Classroom**

Video Clip 1.3, Fifth & Sixth Grade Teacher Interview on Lesson Planning, shows the thinking process that Ms. Bertolone-Smith and Ms. Malone engaged in as they planned the lesson. They chose a rich multi-step problem (see the study guide at the end of this chapter) that required students to make drawings to visualize the problem and represent their thinking. They anticipated the possible errors and misconceptions that students might make.
Their long-term mathematical goal involved teaching students to engage in problem solving and mathematical reasoning and to use schematic drawings as a part of their problem-solving process.

**Planning the Discussion During the Lesson**

A problem-solving approach to teaching is necessary for discussion. In Video Clip 1.3, the teacher starts the lesson by posing a problem, giving students time to think about the problem, then meeting with partners or small groups to further explore the problem. During this small group time, the teacher monitors student understanding by asking probing questions and evaluating work. In this phase of the lesson, the teacher makes a quick decision about what to talk about during the whole class discussion based on student reasoning.

Ms. Bertolone-Smith starts the lesson by introducing a multi-step problem on the interactive white board and having students read it aloud:

A picture 3 feet across hangs in the center of a wall that is 25 feet long. How many feet from the left end of the wall is the left edge of the picture?

Ms. B: I want you to picture a wall with a painting in the center. Now we are going to use a schematic drawing to find the answer to the problem.

As students work with partners and small groups and use their math journals to record their thinking, the teachers walk around, observing what the students are doing and posing questions.

**The Whole Class Discussion: Facilitating Mathematical Connections**

After students have individually thought about a problem and shared ideas with partners or a small group, a whole class discussion takes place.

**A Peek Inside the Classroom**

Video Clip 1.4, Fifth & Sixth Grade Problem-Solving Lesson, shows the whole class discussion that took place in Ms. Bertolone-Smith and Ms. Malone’s classroom.

Students face the interactive white board as Ms B calls for answers and records responses: 11, 10, 12,
In a class of 57 students, the range of five different answers represents five different ways of thinking about the problem. These answers become the focus of discussion as opposed to one student’s thinking. Now, the solutions that yielded the answers become a class issue for discussion. Ms. Bertolone-Smith asks for a volunteer to defend his answer.

**Phase 1: Making Thinking Explicit.** Students explain how they arrived at their answers. Brandon explains his thinking as he displays his drawing, shown here as Figure 1.1, and draws on the interactive white board.

Brandon draws one wall and visualizes the rectangle in the middle as representing the 3-foot painting. He subtracts 3 feet from 25 feet to figure out how much wall space is available that is not covered by the painting. He then divides the free wall space by two and concludes that the answer should be 11.

He accidentally writes the division number sentence backward. When the teacher asks him to check his work, he is able to self-correct.

**Phase 2: Analyzing Each Other’s Solutions.** Brinn shares her solution and explains how it is similar to and different from Brandon’s solution. She came up with the same answer as Brandon but visualized the problem differently.

Brinn’s visualization of the problem is different than Brandon’s, as you can see in Figure 1.2. She viewed the painting as being placed in the center of a 25-foot wall. She figured out the center of the wall and subtracted half the painting width to find the answer.
After Brinn explains her thinking, the teacher asks students to think about their answers and asks students if anyone wants to change their original answers. By doing so, the teacher communicates to the group that not only should they be listening to other students’ answers, but they should also be reflecting if their answer makes sense. Students are expected to reflect on what they see and hear with their own thinking processes.

Nancy: I would like to change my answer from 11.5 to 11. (Another student also decides to change her answer from 12.5 to 11.)

Teacher: Can you tell me why you changed your answer? Tell us why you changed your answer from 12½ to 11.

Student: If the picture frame takes up 3 feet of the wall, then you subtract 3 feet from 25.

Teacher: What did you do before that caused you to arrive at your original answer?

Student: I did not subtract the picture frame from the wall.

Teacher: You did not subtract the 1½ from the 12.5, which got you to the 11. Is that right?

Student: Yeah.

In this example, the students share their thinking and reflect on their own answers. The student could explain how he got a different answer of 12.5. In addition, he could provide a reason for changing his answer based on listening to other students’ explanations. The teacher restated what the student said and asked for clarification. The teachers encouraged students to change their answers based on the new insights they gained.

Misconceptions and errors can be addressed through discussion. The whole class discussion gives students opportunities to discuss why something does or does not make sense. For example, the teacher calls on Ethan to
present his alternate way to think about the problem and why he thinks the answer is 10 feet. Ethan comes up to the interactive board to present his solution (see Figure 1.3). He uses the grid squares to represent one foot in his schematic drawing and counts out 25 squares. Next, he finds the center of the wall is 12.5 squares from the left edge, and he splits the center square in half. He then proceeds to figure out where the edge of the picture frame is by counting 1.5 squares from the center point of his diagram. His answer represents the number of feet from the edge of the picture frame to the end of the left wall.

**Teacher:** Does anyone have any questions for Ethan about his drawing?

Ethan, take a couple of questions from people who maybe don’t understand your thinking clearly. (Several students raise hands.)

Wyatt, go up there and ask him some questions that might be helpful.

Wyatt points to the rectangle representing the picture and indicates that the picture is in the middle of the room. He asks Ethan if he is counting the entire length of the box (3 feet) or if he is counting from the middle of the picture to the end (1.5). By doing so, Wyatt asks Ethan to clarify his thinking.

Ethan explains his drawing by pointing out that each length of a square represents one foot. His 25 boxes represent 25 feet, which is the length of the room. He does not quite understand what Wyatt is asking. Brandon raises his hand and points out that he thinks that Ethan has accidentally not drawn the box in the center of the room. He points out that there are 12 boxes on the right hand side of the box. The teacher asks Brandon to go up there and explain what he means. Brandon walks to the interactive white board to share his observation.

**Teacher:** So, how many feet have we used if we have 10 + 3 + 12? Do we get 25 feet?

Ethan looks as his drawing and changes his mind. After reflecting on Brandon’s reasoning, Ethan reworks the problem and arrives at the correct answer.

**Phase 3: Developing New Mathematical Insights.** A discussion must lead to development of “big mathematical ideas” and skills that students can transfer to solve other problems. Therefore, at the end of the lesson the teacher and students should summarize what they learned. In Video Clip 1.4, students describe what they learned and strategies they can use to solve similar problems. In addition, they also report how to be successful math students. The students say that whole class discussions help them learn to persevere if
they make a mistake, and the importance of checking work and being flexible. They share comments like “don’t just give up, keep trying, check over your work, and keep trying until you get it right.” Their comments, as listed below, show growth in understanding mathematical ideas, representations as an aid to learning, and a change in disposition, in which they see themselves as learners.

The big idea is that you have to remember to find both sides. You have to . . . minus both sides. (mathematical ideas)

Be really exact with drawings; make it straight. (representations)

It is okay to get a wrong answer but don’t be stuck with that and think you have the right answer. It is okay if you figure out a mistake on the board. It is always good to check your work after you figured it out. (disposition)

Video Clip 1.4 illustrates that communication takes many forms. The partner talk, the small group discussions, and the use of drawings as representations to support and communicate thinking all contributed to the whole class discussion. The students’ comments illustrate that not only did they deepen their understanding of how to solve similar problems but they also developed skills needed to study mathematics.

SELF-REFLECTION QUESTIONS

1. What should the teacher’s role in the discussion be?

2. How can you encourage students to do the thinking instead of relying on you for answers?

3. How can you use a range of students’ answers to gauge their understanding?

When looking at these transcripts and the video, examine the nature of the conversation. What was discussed? How did this discussion take place? What was the teacher’s role? What was the students’ role? Did new mathematical understandings develop? Was there value in discussing these types of reasoning and issues as a whole class?
Additional Considerations for Facilitating Effective Discussions

Whole class discussions are much easier to facilitate if the physical space in the classroom has been set up for discussion. In addition, the teacher also needs to cultivate an appropriate social environment for discussion by establishing routines that make discussion easier. Chapter 2 provides ideas on how to set up the classroom space. Chapter 3 explains how to cultivate the social environment for discussion. For example, Ms. Bertolone-Smith and Ms. Malone actively work on creating a classroom culture for discussion. “We set up an environment where it is unacceptable for everyone not to participate.” They want students to actively talk to each other, stay mentally engaged, and challenge each other’s thinking during the math lesson. This expectation is strongly communicated to students. For example, the teachers actively work to make sure the conversations between partners/small groups are meaningful and challenging. If the students are not communicating effectively, they are moved to a different partner. In other words, the teachers maintain flexible groups all the time. Establishing a classroom environment for discussion requires active thought and development throughout the school year.

Chapter Summary

This book focuses on the type of communication covered in the Common Core State and NCTM standards. The purpose of the whole class discussion is to support students’ developmental, conceptual, and procedural understanding of mathematics through problem solving, reasoning, communication, and sense-making. An effective discussion includes three phases: (1) making thinking explicit, (2) analyzing each other’s solutions, and (3) developing new mathematical insights. Whole class discussions contribute to efficient use of class time to support student learning.
Below is a suggested sequence of activities designed to help you apply whole class discussions in your classroom and to think about how whole class discussions can support mathematical learning.

**Examine Discussion Supported Mathematical Learning**

1. Watch the Video Clips outlined in the chapter and review the *Reflecting on Video Clips* questions.
2. Think about how whole class discussion can support mathematical learning.
3. Read the Video Case Study on the next page.

**Try Ideas Out!**

1. Once you have thought about how whole class discussions support learning, try some of the ideas listed in *Strategies for Your Classroom*.
2. Complete the *Reflecting on Practice* worksheet. If you are a pre-service teacher, you can observe a classroom and use your observations to think about the questions provided in the *Reflecting on Practice* worksheet.

**What Makes an Effective or Ineffective Discussion?**

Reflect on what makes an effective and ineffective discussion. Thinking about your own classroom discussions and observations you have made while watching someone else’s discussions are great places to begin thinking about what makes effective discussions. Consider the aspects of the *Reflecting on Practice* worksheet and the Video Clips to further examine how classroom discussion supports mathematical learning.

**Tool Box: PDToolkit**

- *Strategies for Your Classroom: How and When to Use Whole Class Discussion to Support Learning*
- Videos Clips 1.1, 1.2, 1.3, & 1.4
- *Reflecting on Video Clips*
- *Reflecting on Practice: Using Whole Class Discussions*
- PowerPoint: Chapter 1—Why Whole Class Discussions?
VIDEO CASE STUDY

Problem-Solving Using Schematic Drawings

Class: Grades 5 and 6 / Teachers: Ms. Bertolone-Smith and Ms. Malone

The Lesson

Ms. Claudia Bertolone-Smith and Ms. Marlene Malone team-teach math in a combined fifth and sixth grade class. They have approximately 57 students. Minden Elementary School, located in Nevada, is a Title 1 school. This video case shows Ms. Bertolone-Smith and Ms. Malone team-teaching a lesson based on students solving a rich, multi-step problem. The teaching objective is to have students use schematic drawings to support their thinking, and to make the use of schematic drawings part of the problem-solving process.

Problem 1: A picture 3 feet across hangs in the center of a wall that is 25 feet long. How many feet from the left end of the wall is the left edge of the picture? (Kalman, 2008)*

Video Clip 1.4: Fifth & Sixth Grade Class: Problem-Solving Lesson (0:13:52)

Before you watch Video Clip 1.4, solve Problem 1. This will help you follow the students’ explanations. Think about how to represent the problem.

1. What strategies can be used to solve the problem?
2. What do you think students might do?
3. What misconceptions might students have?

Watch the video and think about the following questions:

1. Did the students gain new mathematical insights as a result of this discussion? Why or why not?
2. Did students make connections between each other’s explanations? If so, how did this happen?
3. What caused the student to self-correct his answer?
4. Where in the video do each of the three different phases that are listed in Table 1.2 occur?
5. What was the role of the teacher?
6. How did the students participate in this discussion?
7. Did learning take place?

How and When to Use Whole Class Discussion to Support Learning

Use whole class discussion to:

- Introduce new mathematical ideas
- Address misconceptions and errors
- Help students make deeper mathematical connections
- Help students develop more efficient strategies

Whole class discussion should:

- Focus on sense-making, reasoning and communication
- Build on small group and partner talk
- Address needs of diverse learners
- Help students develop new mathematical insights
## Reflecting on Video Clips

### Video Clip 1.1: Student Interview: Student Perspectives on “Math Talk” and Math Learning

1. What are these students’ perspectives on how “math talk” helps them learn math?
2. What do these students say are the differences between partner talk and whole class discussion?
3. What kinds of experiences related to “math talk” have these students had in their math class?
4. What insights did you gain listening to these students talk about math?

### Video Clip 1.2: Fifth & Sixth Grade Teacher Perspectives on Class Discussion

1. What are the teachers’ mathematical goals for this lesson? Why did they choose these goals? How do these goals contribute to sense-making and discussion?
2. How did the teachers create the classroom environment for discussion with partners?
3. What are the teachers’ perspectives about the value of using partner and small group discussions to teach math? Would you consider using different combinations of math talk in the classroom? Why or why not?
4. What are the teachers’ perspectives on the role of whole class discussions as a means of supporting mathematical learning?

### Video Clip 1.3: Fifth & Sixth Grade Teacher Interview on Lesson Planning

1. What are the teachers’ perspectives on lesson planning?
2. What kinds of lessons promote high level thinking and discussion?
3. What insights did you gain as a result of watching this video?

### Video Clip 1.4: Fifth & Sixth Grade Problem-Solving Lesson

*Note: Before you watch Video Clip 1.4, solve Problem 1, found in the Video Case Study section. This will help you follow the students’ explanations. Think about how to represent the problem.*

1. What strategies can be used to solve the problem?
2. What do you think students might do?
3. What misconceptions might students have?

**Now, watch the video and think about the following questions:**

1. Did the students gain new mathematical insights as a result of this discussion? Why or why not?
2. Did students make connections between each other’s explanations? If so, how did this happen?
3. What caused the student to self-correct his answer?
4. Where in the video do each of the three different phases that are listed in Table 1.2 occur?
5. What was the role of the teacher?
6. How did the students participate in this discussion?
7. Did learning take place?
CHAPTER 1

REFLECTING ON PRACTICE

Using Whole Class Discussions

1. How do you use whole class discussions in your math lessons?
2. What plan is there for whole class discussions?
3. What are some challenges to implementing effective whole class discussions?
4. What are some of your areas of strengths and weaknesses in facilitating discussions?
5. What are some areas of facilitating discussions that you would like to work on?
Designing the Physical Classroom Space for Discussions

Best discussions take place where there is a spirit of inquiry and environment of trust.

(Spiegel, 2005)

As mentioned in Chapter 1, a well-designed classroom space enhances discussions. This chapter explores how to design the physical layout of the classroom and organize classroom spaces for both small group and whole class discussions. Students should be able to easily move to these locations and have access to the tools and resources they need for discussions. In the classroom shown below, students are seated on the floor and using their notebooks to record information about the problem they are working on. All the students can see the easel and teacher, which is one of the essential elements for designing a classroom that facilitates discussion.

**Design Elements for Classroom Space**

There are several essential classroom design elements to consider when setting up your classroom to facilitate whole class discussion. Organizing the classroom’s physical space for effective discussions means creating a focal point, having a place for students and the teacher to gather comfortably, giving easy access to tools.
and materials needed during discussions, and reserving a space to display student work. These elements are illustrated in Figure 2.1 above.

It is important to visualize how these elements can be incorporated into an actual classroom. Figure 2.2 is an example of how one teacher organized her space for whole class discussion. Let’s discuss how she uses the physical space of her classroom to implement the elements in Figure 2.1.

First, students are physically gathered together on the carpet, which is set up to clearly indicate that it is the designated space...
for whole class discussions. Students also bring their math journals to the carpet area so they can refer to the work from their partner or small group discussions when explaining or justifying their thinking. The whole class can see the easel, on which the teacher has written “$\frac{2}{3} = \frac{6}{9}$, true or false?” The teacher is seated behind the students where she can also see the focal point of the discussion. Notice that a student is moving forward to share her ideas about the concept of equivalent fractions and to explain that $\frac{2}{3}$ is equal to $\frac{6}{9}$. The other students listen intently to her explanation. Another student has already written his explanation on the chart paper that is on the easel. You can see the chart paper from the previous day’s discussion is displayed on the bulletin board next to the easel.

The way that the space is organized influences the kinds of interactions that can take place during a discussion. The chart paper easel creates a focal point for students to look at during the conversation. The carpet space becomes an intimate setting for students to gather together. The chart paper on the board displaying student work makes it possible to refer to previous discussions and solutions. Once the physical space is prepared, it is important to make sure that it is functional as well as comfortable.

**SELF-REFLECTION QUESTION**

How does the space in your classroom contribute to small group and whole class discussions?

**Designing a Functional Space for Discussions**

At the beginning of the school year, teachers prepare their classrooms by making decisions about the arrangement of student desks, the teacher desk, materials, and the use of space for activities and meetings. Decisions are influenced by the size of the room, the number of students, and the furniture available.

In addition to creating a warm and inviting environment, teachers need to consider the *functionality* of the space. Functionality involves thinking about how the space is going to be used for the variety of work the class will be doing. One of the most important things to consider is how students can move from space to space to get their work done. Because students will work in small groups, there must be spaces for them to meet, such as the
Designing the Physical Classroom Space for Discussions

In addition, students must be able to easily move from their small groups to a designated space for whole class discussions. Classroom transitions become smoother if the space is arranged so that students can efficiently navigate from one space and activity to the other. Take into consideration the ease of access students have to tools and resources they will use to communicate in the various spaces. Once the space is organized, students will be able to familiarize themselves with which spaces are designated for certain types of activities and how to quickly navigate the physical space between small group discussion and whole class discussion.

Designing Space for Small Group Discussion

When small groups gather, they need a common work space that allows them to face each other. Being face to face makes it easier for them to communicate. Students also need to have access to the tools and resources needed for the lesson. You can either group individual desks together so that students face each other, or use larger desks that can seat several students. Make sure that you are able to walk around the groups to monitor the discussions and observe students. You will use these observations to plan your whole class discussion.

Designing a Functional Space for Whole Class Discussions

Design elements that create a functional space for whole class discussions include a focal point, seating arrangements without distractions, and access to tools and materials. As discussed earlier, use available space to create a separate area for whole class discussions. Students (particularly younger students) are more focused on the discussion if they are physically gathered
close together. This minimizes distractions and makes it easier to make eye contact. Close proximity also helps teachers observe students’ facial expressions and body language. These are important physical cues that identify which students are engaged in the discussion, which students are confused, and which students are truly grasping the lesson.

Figure 2.4 provides an example of how one teacher has organized her classroom space for whole class discussions. The teacher used an area rug to create a separate space for whole class discussion. The rug is placed in front of the white board, between extra seating and storage units. This arrangement clearly structures where students sit for whole class discussion by offering extra definition to the discussion area. Placing a chart paper easel in front of the white boards allows the students to use the chart paper easel and the white board space as needed. In addition, an overhead projector is also available close to the focal point, which gives the teacher easy access to another teaching tool without moving away from the focal point.

Creating a Focal Point

A focal point is the part of the whole group discussion area at which students will be looking during the whole class discussion, as pictured in Figure 2.5. Because the focal point needs to be the center of their attention, it is imperative that all the students can see what is being demonstrated from where they are sitting. The focal point can be a chart paper easel or a white board, but it must be something on which the students can write as they participate in the discussion. The focal point is so important because it is where students will demonstrate their thinking and make comparisons between multiple representations of the problem they are solving. If a student writes a solution to a problem that some students can’t see, the visual representation becomes a useless tool for discussion.

To encourage students to answer questions through discussion, the focal point needs to be the easel or white board, not the teacher. Researchers Nathan
Knuth (2003) found that when the teacher physically removes herself from the focal point and asks questions from the back of the group, the students begin to talk to each other rather than talking directly to the teacher. Students are often willing to take ownership of the conversation when the teacher is not the center of the conversation. However, the topic may require the teacher to stand in front of the group to discuss or scaffold ideas, particularly if the subject is challenging or the discussion is slow to start. Therefore, teachers need to be able to move easily to and from the focal point as needed.

Teachers also need to be able to quickly evaluate the best place to stand for each class. Part of that decision will depend on how dependent the discussion is on the teacher participating in the visual representation of the lesson.

**SELF-REFLECTION QUESTIONS**

1. Where do you usually stand during a classroom lesson?
2. When should you remove yourself from the center to allow students to engage in conversation with each other?

**Visualization and Representations**

Visualization is the ability to form a mental image of an idea or concept. In mathematics, it can be hard to understand what another person is thinking and explaining. However, if students use a model or draw a diagram to support an explanation, it is easier to understand their reasoning. According to the NCTM Principles and Standards for School Mathematics (2000), representations such as
diagrams, graphical displays, and symbolic expressions are an integral part of communication and sense-making.

When a student demonstrates thinking through drawings, pointing, or explanations, it is easier for both the teacher and the other students to visualize and understand the student’s explanation. For example, to figure out how to divide a pizza into fourths, a student might draw a circle and say “I first drew a line like this, and then I did it again like this and then I got 4 pieces.” During the whole class discussion, the student must explain that the circle represents a pizza and the lines represent his divisions to attribute meaning to the representation.

Access to Materials for Students’ Work

Students can use physical tools to represent their ideas and communicate their thinking (NCTM, 2000). In the previous example, the student would need a marker and an easel or white board to draw his “pizza” and divide it. Students need manipulatives, pens, paper, and access to chart paper or a dry erase board. Making these materials accessible near the focal point ensures that the flow of the classroom discussion is not interrupted by students or the teacher leaving the discussion area to retrieve needed materials.

The shelves in Figure 2.6 contain many different kinds of materials that are easily accessible to the teacher and students. By using clear containers, students can quickly see what kinds of tools are available and know where to return them after discussion. To ensure good discussion etiquette, give students easy access to general discussion guidelines and key vocabulary by hanging the information where everyone can see it from the discussion area.

Access to Technology

Technology such as LCD projectors, interactive white boards, document cameras, class websites, and other computer software enhances teaching and learning mathematics by providing students with representations that are difficult to do by hand (NCTM, 2000). Technology can be effectively integrated as part of discussions if it is located within the focal

FIGURE 2.6
Easily accessible materials for student use during discussions
point or easily accessible during the discussion. Technology may eliminate the need to create and store chart paper records. For example, student work can be displayed in a document camera and saved for later use. Students can write on the interactive white board that displays other students’ work to add their thoughts without actually marking the original work. If you have an interactive white board with a document camera, make sure the camera is easily accessible close to the focal point so that students don’t have to travel to a different section of the classroom to retrieve it.

**Displaying and Saving Students’ Work**

We have discussed that it is important to save student work from class discussions, and there are several reasons to do so. Saving student work and having it accessible allows students to return and reference various problem-solving skills, which encourages them to try and apply many concepts when faced with a challenging problem.

Class length is another reason to have a system for saving and displaying student work. If the bell rings before you have finished your whole class discussion, student work should be saved so that it can be easily accessed the next day. One teacher uses a clothes rack with pants hangers to save the work students have done on chart paper, as illustrated in Figure 2.7. Students can access the coat rack during small group discussion, whole class discussion, or seat work. Easy access to previously created ideas allows discussions to continue smoothly.

Unfortunately, this easy access to earlier work is not the norm. More often, students immediately erase their drawings after they explain their thinking during whole class discussions. Consequently, erasing work prevents students from making comparisons between each other’s ideas or revisiting ideas. Storage systems like...
Further Suggestions for Classroom Layouts

The diagrams in Figure 2.8 provide examples of classroom arrangements that facilitate whole class discussions. The large rectangle in Figure 2.8a and large circle in Figure 2.8b represent carpeted areas. In all three diagrams, there is a clear focal point. Figures 2.8a and b show classrooms with separate carpet space and focal point for discussion. Teachers have reported that a separate carpet space helps students stay focused because distractions are minimized. However, if the students are older and there is limited classroom space, furniture could be arranged so that students can face the focal point with minimal distractions, as shown in Figure 2.8c.

Classrooms often begin the year with a well-designed space. However, material accumulates as the year progresses, and the space becomes disorganized. Therefore, no matter how the physical layout of the classroom is designed, space must periodically be examined and de-cluttered in order to maintain an efficient classroom.

Figure 2.8
Arrangements of classroom space for whole class discussions
Chapter Summary

A whole class discussion is much easier to facilitate when the physical space of the classroom is designed with discussions in mind. The classroom needs to be designed for both small group and whole class discussion. For whole class discussions, there needs to be space for students to gather around the focal point, which is where students look during the discussion to display their work and to analyze multiple representations. Materials and tools for discussion must be readily available so that the flow of conversation is not interrupted. In addition, the teacher must think about how to save student work for future reference. A well-designed physical space optimizes the quality of discussion by saving time and contributing to a smooth flow of the conversation.
Below is a suggested sequence of activities designed to help you organize your classroom in a way that optimizes your classroom space for discussions.

**Design Optimal Classroom Space for Discussions**

1. Evaluate and explore the design features of your own classroom.
2. Think about how to design your classroom even if you are not teaching yet.

**Practicing Teacher**

1. Review the *Strategies for Your Classroom: Considerations for Creating Physical Space*. If you have your own classroom, evaluate the layout. Think about the design of your classroom for both small group and whole class discussions. Specifically examine functionality of your space for movement, displaying work, and having access to tools needed during math discussions.
2. Use the *Physical Layouts for Classroom Discussion Checklist* to guide you while you examine your space for design and functionality that facilitates discussions.
3. Ask a colleague for feedback on your redesigned space. It is helpful to give your colleague a blank copy of *Physical Layouts for Classroom Discussion Checklist*. It will guide them to review your changes in a more concrete way. Having another person evaluate your space may give you a fresh perspective or a layout idea you overlooked.

**Pre-Service Teacher**

1. If you do not have your classroom just yet, try to imagine how you would like to organize your space. You can also use classrooms you have observed in the past, and think about how you would modify that organization based on the *Physical Layouts for Classroom Discussion Checklist*.
2. Draw a diagram of your room on the *Reflecting on Practice* worksheet.
3. Have peers evaluate your layout. It may helpful to give your colleague a blank copy of *Physical Layouts for Classroom Discussion Checklist*. Using these tools offers an opportunity for further discussion of classroom types and layout options that could optimize whole class discussion.

**Tool Box: PDToolkit**

- *Strategies for Your Classroom: Considerations for Creating Physical Space*
- *Reflecting on Practice: Designing Classroom Space*
- *Physical Layouts for Classroom Discussions Checklist*
- PowerPoint: Chapter 2—Designing the Physical Classroom Space for Discussion
Considerations for Creating Physical Space

**Small Group Discussion Space**
- Create a common space to work where students can see each other
- Have space for the teacher to walk around and observe student conversations
- Set clear paths for students to transition to the whole group space (such as a carpet)

**Large Group Discussion Space**
- Implement a point that can be viewed by all students (e.g., white board, interactive board, easel with chart paper)
- Reserve a place to display multiple student work
- Designate space for students to sit facing the focal point
- Clear a path for students to walk back and forth to the focal point while sharing answers
- Have a place for the teacher to stand both in front of and behind the group during the discussion
- Create a storage place for materials and supplies that is close to the focal point
- Have access to technology (e.g., doc cam and interactive boards)
- Create a system for storing student work for future reference (either physically or digitally)
Designing Classroom Space

1. Draw a diagram of how you would organize your classroom space.

2. Describe the functional features of your space.

3. What technology and tools do you have available in your class for discussion? Are they located in a place with easy access during the discussion?
# Physical Layouts for Classroom Discussions Checklist

Use this checklist to evaluate the design and functionality of your classroom space for discussions. Once you have made your changes, ask a colleague or math coach to provide you with their impressions.

<table>
<thead>
<tr>
<th>Observed</th>
<th>Notes—To-do list</th>
</tr>
</thead>
</table>

## Small Group
- Is there space for small group discussion?
- Is there space for the teacher to walk around and observe the students?
- Are there paths for students to walk to the whole group space?

## Whole Group
- Is the focal point visible to the whole group?
- Does the focal point have a place to write ideas and representations?
- Is there a place for students to sit?
- Is there a place for the teacher to stand/sit both in front of and away from the focal point?
- Do you have access to technology for discussions?
- Do you have easy access to materials and supplies for discussion?
- Is there a place to share multiple student work?
- Do you have a place to store student work (either physically or digitally)?

## Overall Effectiveness
- Areas that need improvement
- Areas that need de-cluttering