Fluid Mechanics for Chemical Engineers

Second Edition

with Microfluidics and CFD
The Prentice Hall International Series in the Physical and Chemical Engineering Sciences had its auspicious beginning in 1956 under the direction of Neal R. Amundsen. The series comprises the most widely adopted college textbooks and supplements for chemical engineering education. Books in this series are written by the foremost educators and researchers in the field of chemical engineering.

Visit informit.com/ph/physandchem for a complete list of available publications.
Dedicated to the memory of

Terence Robert Corelli Fox

Shell Professor of Chemical Engineering
University of Cambridge, 1946–1959
This page intentionally left blank
PART I—MACROSCOPIC FLUID MECHANICS

CHAPTER 1—INTRODUCTION TO FLUID MECHANICS

1.1 Fluid Mechanics in Chemical Engineering 3
1.2 General Concepts of a Fluid 3
1.3 Stresses, Pressure, Velocity, and the Basic Laws 5
1.4 Physical Properties—Density, Viscosity, and Surface Tension 10
1.5 Units and Systems of Units 21
   Example 1.1—Units Conversion 24
   Example 1.2—Mass of Air in a Room 25
1.6 Hydrostatics 26
   Example 1.3—Pressure in an Oil Storage Tank 29
   Example 1.4—Multiple Fluid Hydrostatics 30
   Example 1.5—Pressure Variations in a Gas 31
   Example 1.6—Hydrostatic Force on a Curved Surface 35
   Example 1.7—Application of Archimedes’ Law 37
1.7 Pressure Change Caused by Rotation 39
   Example 1.8—Overflow from a Spinning Container 40
Problems for Chapter 1 42

CHAPTER 2—MASS, ENERGY, AND MOMENTUM BALANCES

2.1 General Conservation Laws 55
2.2 Mass Balances 57
   Example 2.1—Mass Balance for Tank Evacuation 58
2.3 Energy Balances 61
   Example 2.2—Pumping \( n \)-Pentane 65
2.4 Bernoulli’s Equation 67
2.5 Applications of Bernoulli’s Equation 70
   Example 2.3—Tank Filling 76
2.6 Momentum Balances 78
   Example 2.4—Impinging Jet of Water 83
   Example 2.5—Velocity of Wave on Water 84
   Example 2.6—Flow Measurement by a Rotameter 89
2.7 Pressure, Velocity, and Flow Rate Measurement 92
Problems for Chapter 2 96

CHAPTER 3—FLUID FRICTION IN PIPES

3.1 Introduction 120
3.2 Laminar Flow 123
   Example 3.1—Polymer Flow in a Pipeline 128
3.3 Models for Shear Stress 129
3.4 Piping and Pumping Problems
   Example 3.2—Unloading Oil from a Tanker
   Specified Flow Rate and Diameter 142
   Example 3.3—Unloading Oil from a Tanker
   Specified Diameter and Pressure Drop 144
   Example 3.4—Unloading Oil from a Tanker
   Specified Flow Rate and Pressure Drop 147
   Example 3.5—Unloading Oil from a Tanker
   Miscellaneous Additional Calculations 147
3.5 Flow in Noncircular Ducts 150
   Example 3.6—Flow in an Irrigation Ditch 152
3.6 Compressible Gas Flow in Pipelines 156
3.7 Compressible Flow in Nozzles 159
3.8 Complex Piping Systems
   Example 3.7—Solution of a Piping/Pumping Problem 165
Problems for Chapter 3 168

CHAPTER 4—FLOW IN CHEMICAL ENGINEERING EQUIPMENT

4.1 Introduction 185
4.2 Pumps and Compressors 188
   Example 4.1—Pumps in Series and Parallel 193
4.3 Drag Force on Solid Particles in Fluids 194
   Example 4.2—Manufacture of Lead Shot 202
4.4 Flow Through Packed Beds
   Example 4.3—Pressure Drop in a Packed-Bed Reactor 208
4.5 Filtration 210
4.6 Fluidization 215
4.7 Dynamics of a Bubble-Cap Distillation Column 216
4.8 Cyclone Separators 219
4.9 Sedimentation 222
4.10 Dimensional Analysis
   Example 4.4—Thickness of the Laminar Sublayer 229
Problems for Chapter 4 230
PART II—MICROSCOPIC FLUID MECHANICS

CHAPTER 5—DIFFERENTIAL EQUATIONS OF FLUID MECHANICS

5.1 Introduction to Vector Analysis ................................................. 249
5.2 Vector Operations .................................................................. 250
  Example 5.1—The Gradient of a Scalar ...................................... 253
  Example 5.2—The Divergence of a Vector .................................. 257
  Example 5.3—An Alternative to the Differential Element .......... 257
  Example 5.4—The Curl of a Vector .......................................... 262
  Example 5.5—The Laplacian of a Scalar .................................... 262
5.3 Other Coordinate Systems ...................................................... 263
5.4 The Convective Derivative ...................................................... 266
5.5 Differential Mass Balance ...................................................... 267
  Example 5.6—Physical Interpretation of the Net Rate of Mass Outflow 269
  Example 5.7—Alternative Derivation of the Continuity Equation .... 270
5.6 Differential Momentum Balances ........................................... 271
5.7 Newtonian Stress Components in Cartesian Coordinates ........ 274
  Example 5.8—Constant-Viscosity Momentum Balances in Terms of Velocity Gradients 280
  Example 5.9—Vector Form of Variable-Viscosity Momentum Balance 284
Problems for Chapter 5 ............................................................... 285

CHAPTER 6—SOLUTION OF VISCOUS-FLOW PROBLEMS

6.1 Introduction ........................................................................... 292
6.2 Solution of the Equations of Motion in Rectangular Coordinates ......................................................... 294
  Example 6.1—Flow Between Parallel Plates ............................. 294
6.3 Alternative Solution Using a Shell Balance ......................... 301
  Example 6.2—Shell Balance for Flow Between Parallel Plates .... 301
  Example 6.3—Film Flow on a Moving Substrate .................... 303
  Example 6.4—Transient Viscous Diffusion of Momentum (COMSOL) 307
6.4 Poiseuille and Couette Flows in Polymer Processing ................. 312
  Example 6.5—The Single-Screw Extruder ............................... 313
  Example 6.6—Flow Patterns in a Screw Extruder (COMSOL) .... 318
8.7 Boundary-Layer Separation
Example 8.3—Boundary-Layer Flow Between Parallel Plates (COMSOL Library) 435
Example 8.4—Entrance Region for Laminar Flow Between Flat Plates 440
8.8 The Lubrication Approximation
Example 8.5—Flow in a Lubricated Bearing (COMSOL) 448
8.9 Polymer Processing by Calendering
Example 8.6—Pressure Distribution in a Calendered Sheet 454
8.10 Thin Films and Surface Tension
Problems for Chapter 8 459

CHAPTER 9—TURBULENT FLOW
9.1 Introduction
Example 9.1—Numerical Illustration of a Reynolds Stress Term 479
9.2 Physical Interpretation of the Reynolds Stresses 480
9.3 Mixing-Length Theory 481
9.4 Determination of Eddy Kinematic Viscosity and Mixing Length 484
9.5 Velocity Profiles Based on Mixing-Length Theory
Example 9.2—Investigation of the von Kármán Hypothesis 487
9.6 The Universal Velocity Profile for Smooth Pipes 488
9.7 Friction Factor in Terms of Reynolds Number for Smooth Pipes
Example 9.3—Expression for the Mean Velocity 491
9.8 Thickness of the Laminar Sublayer 492
9.9 Velocity Profiles and Friction Factor for Rough Pipe 494
9.10 Blasius-Type Law and the Power-Law Velocity Profile 495
9.11 A Correlation for the Reynolds Stresses 496
9.12 Computation of Turbulence by the $k/\varepsilon$ Method
Example 9.4—Flow Through an Orifice Plate (COMSOL) 501
Example 9.5—Turbulent Jet Flow (COMSOL) 505
9.13 Analogies Between Momentum and Heat Transfer
Example 9.6—Evaluation of the Momentum/Heat-Transfer Analogies 511
9.14 Turbulent Jets
Problems for Chapter 9 521
CHAPTER 10—BUBBLE MOTION, TWO-PHASE FLOW, AND FLUIDIZATION

10.1 Introduction 531
10.2 Rise of Bubbles in Unconfined Liquids 531
   Example 10.1—Rise Velocity of Single Bubbles 536
10.3 Pressure Drop and Void Fraction in Horizontal Pipes 536
   Example 10.2—Two-Phase Flow in a Horizontal Pipe 541
10.4 Two-Phase Flow in Vertical Pipes 543
   Example 10.3—Limits of Bubble Flow 546
   Example 10.4—Performance of a Gas-Lift Pump 550
   Example 10.5—Two-Phase Flow in a Vertical Pipe 553
10.5 Flooding 555
10.6 Introduction to Fluidization 559
10.7 Bubble Mechanics 561
10.8 Bubbles in Aggregatively Fluidized Beds 566
   Example 10.6—Fluidized Bed with Reaction (C) 572
Problems for Chapter 10 575

CHAPTER 11—NON-NEWTONIAN FLUIDS

11.1 Introduction 591
11.2 Classification of Non-Newtonian Fluids 592
11.3 Constitutive Equations for Inelastic Viscous Fluids 595
   Example 11.1—Pipe Flow of a Power-Law Fluid 600
   Example 11.2—Pipe Flow of a Bingham Plastic 604
   Example 11.3—Non-Newtonian Flow in a Die
      (COMSOL Library) 606
11.4 Constitutive Equations for Viscoelastic Fluids 613
11.5 Response to Oscillatory Shear 620
11.6 Characterization of the Rheological Properties of Fluids 623
   Example 11.4—Proof of the Rabinowitsch Equation 624
   Example 11.5—Working Equation for a Coaxial-
      Cylinder Rheometer: Newtonian Fluid 628
Problems for Chapter 11 630

CHAPTER 12—MICROFLUIDICS AND ELECTROKINETIC FLOW EFFECTS

12.1 Introduction 639
12.2 Physics of Microscale Fluid Mechanics 640
12.3 Pressure-Driven Flow Through Microscale Tubes 641
   Example 12.1—Calculation of Reynolds Numbers 641
12.4 Mixing, Transport, and Dispersion 642
12.5 Species, Energy, and Charge Transport 644
12.6 The Electrical Double Layer and Electrokinetic Phenomena 647
   Example 12.2—Relative Magnitudes of Electroosmotic and Pressure-Driven Flows 648
   Example 12.3—Electroosmotic Flow Around a Particle 653
   Example 12.4—Electroosmosis in a Microchannel (COMSOL) 653
   Example 12.5—Electroosmotic Switching in a Branched Microchannel (COMSOL) 657
12.7 Measuring the Zeta Potential 659
   Example 12.6—Magnitude of Typical Streaming Potentials 660
12.8 Electroviscosity 661
12.9 Particle and Macromolecule Motion in Microfluidic Channels 661
   Example 12.7—Gravitational and Magnetic Settling of Assay Beads 662
Problems for Chapter 12 666

CHAPTER 13—AN INTRODUCTION TO COMPUTATIONAL FLUID DYNAMICS AND FLOWLAB

13.1 Introduction and Motivation 671
13.2 Numerical Methods 673
13.3 Learning CFD by Using FlowLab 682
13.4 Practical CFD Examples 686
   Example 13.1—Developing Flow in a Pipe Entrance Region (FlowLab) 687
   Example 13.2—Pipe Flow Through a Sudden Expansion (FlowLab) 690
   Example 13.3—A Two-Dimensional Mixing Junction (FlowLab) 692
   Example 13.4—Flow Over a Cylinder (FlowLab) 696
References for Chapter 13 702

CHAPTER 14—COMSOL (FEMLAB) MULTIPHYSICS FOR SOLVING FLUID MECHANICS PROBLEMS

14.1 Introduction to COMSOL 703
14.2 How to Run COMSOL 705
   Example 14.1—Flow in a Porous Medium with an Obstruction (COMSOL) 705
14.3 Draw Mode 719
14.4 Solution and Related Modes 724
Contents

14.5 Fluid Mechanics Problems Solvable by COMSOL 725
Problems for Chapter 14 730

APPENDIX A: USEFUL MATHEMATICAL RELATIONSHIPS 731
APPENDIX B: ANSWERS TO THE TRUE/FALSE ASSERTIONS 737
APPENDIX C: SOME VECTOR AND TENSOR OPERATIONS 740
INDEX 743
THE AUTHORS 753
This text has evolved from a need for a single volume that embraces a wide range of topics in fluid mechanics. The material consists of two parts—four chapters on macroscopic or relatively large-scale phenomena, followed by ten chapters on microscopic or relatively small-scale phenomena. Throughout, I have tried to keep in mind topics of industrial importance to the chemical engineer. The scheme is summarized in the following list of chapters.

**Part I—Macroscopic Fluid Mechanics**

1. Introduction to Fluid Mechanics  
3. Fluid Friction in Pipes  
4. Flow in Chemical Engineering Equipment

**Part II—Microscopic Fluid Mechanics**

5. Differential Equations of Fluid Mechanics  
6. Solution of Viscous-Flow Problems  
7. Laplace’s Equation, Irrotational and Porous-Media Flows  
8. Boundary-Layer and Other Nearly Unidirectional Flows  
9. Turbulent Flow  
10. Bubble Motion, Two-Phase Flow, and Fluidization  
11. Non-Newtonian Fluids  
12. Microfluidics and Electrokinetic Flow Effects  
13. An Introduction to Computational Fluid Dynamics and FlowLab  
14. COMSOL (FEMLAB) Multiphysics for Solving Fluid Mechanics Problems

In our experience, an undergraduate fluid mechanics course can be based on Part I plus selected parts of Part II, and a graduate course can be based on much of Part II, supplemented perhaps by additional material on topics such as approximate methods and stability.

*Second edition.* I have attempted to bring the book up to date by the major addition of Chapters 12, 13, and 14—one on microfluidics and two on CFD (computational fluid dynamics). The choice of software for the CFD presented a difficulty; for various reasons, I selected FlowLab and COMSOL Multiphysics, but there was no intention of “promoting” these in favor of other excellent CFD programs.¹ The use of CFD examples in the classroom really makes the subject

¹ The software name “FEMLAB” was changed to “COMSOL Multiphysics” in September 2005, the first release under the new name being COMSOL 3.2.
come “alive,” because the previous restrictive necessities of “nice” geometries and constant physical properties, etc., can now be lifted. Chapter 9, on turbulence, has also been extensively rewritten; here again, CFD allows us to venture beyond the usual flow in a pipe or between parallel plates and to investigate further practical situations such as turbulent mixing and recirculating flows.

Example problems. There is an average of about six completely worked examples in each chapter, including several involving COMSOL (dispersed throughout Part II) and FlowLab (all in Chapter 13). The end of each example is marked by a small, hollow square: □. All the COMSOL examples have been run on a Macintosh G4 computer using FEMLAB 3.1, but have also been checked on a PC; those using a PC or other releases of COMSOL/FEMLAB may encounter slightly different windows than those reproduced here. The format for each COMSOL example is: (a) problem statement, (b) details of COMSOL implementation, and (c) results and discussion (however, item (b) can easily be skipped for those interested only in the results).

The numerous end-of-chapter problems have been classified roughly as easy (E), moderate (M), or difficult/lengthy (D). The University of Cambridge has given permission—kindly endorsed by Professor J.F. Davidson, F.R.S.—for several of their chemical engineering examination problems to be reproduced in original or modified form, and these have been given the additional designation of “(C)”.

Acknowledgments. I gratefully acknowledge the written contributions of my former Michigan colleague Stacy Birmingham (non-Newtonian fluids), Brian Kirby of Cornell University (microfluidics), and Chi-Yang Cheng of Fluent, Inc. (FlowLab). Although I wrote most of the COMSOL examples, I have had great help and cooperation from COMSOL Inc. and the following personnel in particular—Philip Byrne, Bjorn Sjodin, Ed Fontes, Peter Georen, Olof Hernell, Johan Linde, and Rémi Magnard. At Fluent, Inc., Shane Moeykens was instrumental in identifying Chi-Yang Cheng as the person best suited to write the FlowLab chapter. Courtney Esposito and Jordan Schmidt of The MathWorks kindly helped me with MATLAB, needed for the earlier 2.3 version of FEMLAB.

I appreciate the assistance of several other friends and colleagues, including Nitin Anturkar, Stuart Churchill, John Ellis, Kevin Ellwood, Scott Fogler, Leenaporn Jongpaiboonkit, Lisa Keyser, Kartic Khilar, Ronald Larson, Susan Montgomery, Donald Nicklin, the late Margaret Sansom, Michael Solomon, Sandra Swisher, Rasin Tek, Robert Ziff, and my wife Mary Ann Gibson Wilkes. Also very helpful were Bernard Goodwin, Elizabeth Ryan, and Michelle Housley at Prentice Hall PTR, and my many students and friends at the University of Michigan and Chulalongkorn University in Bangkok. Others are acknowledged in specific literature citations.

Further information. The website http://www.engin.umich.edu/~fmche is maintained as a “bulletin board” for giving additional information about the
book—hints for problem solutions, *errata*, how to contact the authors, etc.—as proves desirable. My own Internet address is wilkes@umich.edu. The text was composed on a Power Macintosh G4 computer using the TEXtures “typesetting” program. Eleven-point type was used for the majority of the text. Most of the figures were constructed using MacDraw Pro, Excel, and KaleidaGraph.

Professor Terence Fox, to whom this book is dedicated, was a Cambridge engineering graduate who worked from 1933 to 1937 at Imperial Chemical Industries Ltd., Billingham, Yorkshire. Returning to Cambridge, he taught engineering from 1937 to 1946 before being selected to lead the Department of Chemical Engineering at the University of Cambridge during its formative years after the end of World War II. As a scholar and a gentleman, Fox was a shy but exceptionally brilliant person who had great insight into what was important and who quickly brought the department to a preeminent position. He succeeded in combining an industrial perspective with intellectual rigor. Fox relinquished the leadership of the department in 1959, after he had secured a permanent new building for it (carefully designed in part by himself).

Fox was instrumental in bringing Peter Danckwerts, Kenneth Denbigh, John Davidson, and others into the department. He also accepted me in 1956 as a junior faculty member, and I spent four good years in the Cambridge University Department of Chemical Engineering. Danckwerts subsequently wrote an appreciation\(^2\) of Fox’s talents, saying, with almost complete accuracy: “Fox instigated no research and published nothing.” How times have changed—today, unless he were known personally, his résumé would probably be cast aside and he would stand little chance of being hired, let alone of receiving tenure! However, his lectures, meticulously written handouts, enthusiasm, genius, and friendship were a great inspiration to me, and I have much pleasure in acknowledging his positive impact on my career.

James O. Wilkes
August 18, 2005

---

<table>
<thead>
<tr>
<th>Greek Letter</th>
<th>English Equivalent</th>
</tr>
</thead>
<tbody>
<tr>
<td>α</td>
<td>alpha</td>
</tr>
<tr>
<td>β</td>
<td>beta</td>
</tr>
<tr>
<td>γ, Γ</td>
<td>gamma</td>
</tr>
<tr>
<td>δ, Δ</td>
<td>delta</td>
</tr>
<tr>
<td>ε, ζ</td>
<td>epsilon</td>
</tr>
<tr>
<td>ζ</td>
<td>zeta</td>
</tr>
<tr>
<td>η</td>
<td>eta</td>
</tr>
<tr>
<td>θ, θ, Θ</td>
<td>theta</td>
</tr>
<tr>
<td>i</td>
<td>iota</td>
</tr>
<tr>
<td>κ</td>
<td>kappa</td>
</tr>
<tr>
<td>λ, Λ</td>
<td>lambda</td>
</tr>
<tr>
<td>μ</td>
<td>mu</td>
</tr>
<tr>
<td>ν</td>
<td>nu</td>
</tr>
<tr>
<td>ξ, Ξ</td>
<td>xi</td>
</tr>
<tr>
<td>ο</td>
<td>omicron</td>
</tr>
<tr>
<td>π, π, Π</td>
<td>pi</td>
</tr>
<tr>
<td>ρ, ρ</td>
<td>rho</td>
</tr>
<tr>
<td>σ, ζ, Σ</td>
<td>sigma</td>
</tr>
<tr>
<td>τ</td>
<td>tau</td>
</tr>
<tr>
<td>υ, ϒ</td>
<td>upsilon</td>
</tr>
<tr>
<td>φ, ϕ, Φ</td>
<td>phi</td>
</tr>
<tr>
<td>χ</td>
<td>chi</td>
</tr>
<tr>
<td>ψ, Ψ</td>
<td>psi</td>
</tr>
<tr>
<td>ω, Ω</td>
<td>omega</td>
</tr>
</tbody>
</table>
1.1 Fluid Mechanics in Chemical Engineering

A knowledge of fluid mechanics is essential for the chemical engineer because the majority of chemical-processing operations are conducted either partly or totally in the fluid phase. Examples of such operations abound in the biochemical, chemical, energy, fermentation, materials, mining, petroleum, pharmaceuticals, polymer, and waste-processing industries.

There are two principal reasons for placing such an emphasis on fluids. First, at typical operating conditions, an enormous number of materials normally exist as gases or liquids, or can be transformed into such phases. Second, it is usually more efficient and cost-effective to work with fluids in contrast to solids. Even some operations with solids can be conducted in a quasi-fluidlike manner; examples are the fluidized-bed catalytic refining of hydrocarbons, and the long-distance pipelining of coal particles using water as the agitating and transporting medium.

Although there is inevitably a significant amount of theoretical development, almost all the material in this book has some application to chemical processing and other important practical situations. Throughout, we shall endeavor to present an understanding of the physical behavior involved; only then is it really possible to comprehend the accompanying theory and equations.

1.2 General Concepts of a Fluid

We must begin by responding to the question, “What is a fluid?” Broadly speaking, a fluid is a substance that will deform continuously when it is subjected to a tangential or shear force, much as a similar type of force is exerted when a water-skier skims over the surface of a lake or butter is spread on a slice of bread. The rate at which the fluid deforms continuously depends not only on the magnitude of the applied force but also on a property of the fluid called its viscosity or resistance to deformation and flow. Solids will also deform when sheared, but a position of equilibrium is soon reached in which elastic forces induced by the deformation of the solid exactly counterbalance the applied shear force, and further deformation ceases.
A simple apparatus for shearing a fluid is shown in Fig. 1.1. The fluid is contained between two concentric cylinders; the outer cylinder is stationary, and the inner one (of radius $R$) is rotated steadily with an angular velocity $\omega$. This shearing motion of a fluid can continue indefinitely, provided that a source of energy—supplied by means of a torque here—is available for rotating the inner cylinder. The diagram also shows the resulting velocity profile; note that the velocity in the direction of rotation varies from the peripheral velocity $R\omega$ of the inner cylinder down to zero at the outer stationary cylinder, these representing typical no-slip conditions at both locations. However, if the intervening space is filled with a solid—even one with obvious elasticity, such as rubber—only a limited rotation will be possible before a position of equilibrium is reached, unless, of course, the torque is so high that slip occurs between the rubber and the cylinder.

There are various classes of fluids. Those that behave according to nice and obvious simple laws, such as water, oil, and air, are generally called Newtonian fluids. These fluids exhibit constant viscosity but, under typical processing conditions, virtually no elasticity. Fortunately, a very large number of fluids of interest to the chemical engineer exhibit Newtonian behavior, which will be assumed throughout the book, except in Chapter 11, which is devoted to the study of non-Newtonian fluids.

A fluid whose viscosity is not constant (but depends, for example, on the intensity to which it is being sheared), or which exhibits significant elasticity, is termed non-Newtonian. For example, several polymeric materials subject to deformation can “remember” their recent molecular configurations, and in attempting to recover their recent states, they will exhibit elasticity in addition to viscosity. Other fluids, such as drilling mud and toothpaste, behave essentially as solids and
will *not* flow when subject to *small* shear forces, but *will* flow readily under the influence of *high* shear forces.

Fluids can also be broadly classified into two main categories—liquids and gases. *Liquids* are characterized by relatively high densities and viscosities, with molecules close together; their volumes tend to remain constant, roughly independent of pressure, temperature, or the size of the vessels containing them. *Gases*, on the other hand, have relatively low densities and viscosities, with molecules far apart; generally, they will rapidly tend to fill the container in which they are placed. However, these two states—liquid and gaseous—represent but the two extreme ends of a continuous *spectrum* of possibilities.

Fluids can also be broadly classified into two main categories—liquids and gases. *Liquids* are characterized by relatively high densities and viscosities, with molecules close together; their volumes tend to remain constant, roughly independent of pressure, temperature, or the size of the vessels containing them. *Gases*, on the other hand, have relatively low densities and viscosities, with molecules far apart; generally, they will rapidly tend to fill the container in which they are placed. However, these two states—liquid and gaseous—represent but the two extreme ends of a continuous *spectrum* of possibilities.

![Fig. 1.2 When does a liquid become a gas?](image)

The situation is readily illustrated by considering a fluid that is initially a gas at point G on the pressure/temperature diagram shown in Fig. 1.2. By increasing the pressure, and perhaps lowering the temperature, the vapor-pressure curve is soon reached and crossed, and the fluid condenses and apparently becomes a liquid at point L. By continuously adjusting the pressure and temperature so that the clockwise path is followed, and circumnavigating the critical point C in the process, the fluid is returned to G, where it is presumably once more a gas. But where does the transition from liquid at L to gas at G occur? The answer is at no single point, but rather that the change is a continuous and gradual one, through a whole spectrum of intermediate states.

### 1.3 Stresses, Pressure, Velocity, and the Basic Laws

**Stresses.** The concept of a *force* should be readily apparent. In fluid mechanics, a force per unit area, called a *stress*, is usually found to be a more convenient and versatile quantity than the force itself. Further, when considering a specific surface, there are two types of stresses that are particularly important.

1. The first type of stress, shown in Fig. 1.3(a), acts *perpendicularly* to the surface and is therefore called a *normal* stress; it will be tensile or compressive, depending on whether it tends to stretch or to compress the fluid on which it acts. The normal stress equals \( F/A \), where \( F \) is the normal force and \( A \) is the area of the surface on which it acts. The dotted outlines show the volume changes caused
by deformation. In fluid mechanics, pressure is usually the most important type of compressive stress, and will shortly be discussed in more detail.

2. The second type of stress, shown in Fig. 1.3(b), acts tangentially to the surface; it is called a shear stress $\tau$, and equals $F/A$, where $F$ is the tangential force and $A$ is the area on which it acts. Shear stress is transmitted through a fluid by interaction of the molecules with one another. A knowledge of the shear stress is very important when studying the flow of viscous Newtonian fluids. For a given rate of deformation, measured by the time derivative $d\gamma/dt$ of the small angle of deformation $\gamma$, the shear stress $\tau$ is directly proportional to the viscosity of the fluid (see Fig. 1.3(b)).

**Fig. 1.3(a)** Tensile and compressive normal stresses $F/A$, acting on a cylinder, causing elongation and shrinkage, respectively.

![Fig. 1.3(a)](image)

**Fig. 1.3(b)** Shear stress $\tau = F/A$, acting on a rectangular parallelepiped, shown in cross section, causing a deformation measured by the angle $\gamma$ (whose magnitude is exaggerated here).

**Pressure.** In virtually all hydrostatic situations—those involving fluids at rest—the fluid molecules are in a state of compression. For example, for the swimming pool whose cross section is depicted in Fig. 1.4, this compression at a typical point P is caused by the downwards gravitational weight of the water above point P. The degree of compression is measured by a scalar, $p$—the pressure.

A small inflated spherical balloon pulled down from the surface and tethered at the bottom by a weight will still retain its spherical shape (apart from a small distortion at the point of the tether), but will be diminished in size, as in Fig. 1.4(a). It is apparent that there must be forces acting normally inward on the
surface of the balloon, and that these must essentially be uniform for the shape to remain spherical, as in Fig. 1.4(b).

![Diagram of balloon submerged in water with pressure forces](image)

**Fig. 1.4** (a) Balloon submerged in a swimming pool; (b) enlarged view of the compressed balloon, with pressure forces acting on it.

Although the pressure \( p \) is a scalar, it typically appears in tandem with an area \( A \) (assumed small enough so that the pressure is uniform over it). By definition of pressure, the surface experiences a normal compressive force \( F = pA \). Thus, pressure has units of a force per unit area—the same as a stress.

The value of the pressure at a point is independent of the orientation of any area associated with it, as can be deduced with reference to a differentially small wedge-shaped element of the fluid, shown in Fig. 1.5.

![Diagram of wedge of fluid](image)

**Fig. 1.5** Equilibrium of a wedge of fluid.

Due to the pressure there are three forces, \( p_A dA \), \( p_B dB \), and \( p_C dC \), that act on the three rectangular faces of areas \( dA \), \( dB \), and \( dC \). Since the wedge is not moving, equate the two forces acting on it in the horizontal or \( x \) direction, noting that \( p_A dA \) must be resolved through an angle \((\pi/2 - \theta)\) by multiplying it by \( \cos(\pi/2 - \theta) = \sin \theta \):

\[
p_A dA \sin \theta = p_C dC. \tag{1.1}
\]

The vertical force \( p_B dB \) acting on the bottom surface is omitted from Eqn. (1.1) because it has no component in the \( x \) direction. The horizontal pressure forces
acting in the $y$ direction on the two triangular faces of the wedge are also omitted, since again these forces have no effect in the $x$ direction. From geometrical considerations, areas $dA$ and $dC$ are related by:

$$dC = dA \sin \theta.$$  \hspace{1cm} (1.2)

These last two equations yield:

$$p_A = p_C,$$  \hspace{1cm} (1.3)

verifying that the pressure is independent of the orientation of the surface being considered. A force balance in the $z$ direction leads to a similar result, $p_A = p_B$.\(^1\)

For *moving* fluids, the normal stresses include both a pressure *and* extra stresses caused by the motion of the fluid, as discussed in detail in Section 5.6.

The amount by which a certain pressure exceeds that of the atmosphere is termed the *gauge* pressure, the reason being that many common pressure gauges are really differential instruments, reading the difference between a required pressure and that of the surrounding atmosphere. *Absolute* pressure equals the gauge pressure plus the atmospheric pressure.

**Velocity.** Many problems in fluid mechanics deal with the *velocity* of the fluid at a point, equal to the rate of change of the position of a fluid particle with time, thus having both a magnitude and a direction. In some situations, particularly those treated from the *macroscopic* viewpoint, as in Chapters 2, 3, and 4, it sometimes suffices to ignore variations of the velocity with position. In other cases—particularly those treated from the *microscopic* viewpoint, as in Chapter 6 and later—it is invariably essential to consider variations of velocity with position.

![Fig. 1.6 Fluid passing through an area A: (a) Uniform velocity, (b) varying velocity.](image)

Velocity is not only important in its own right, but leads immediately to three *fluxes* or flow rates. Specifically, if $u$ denotes a *uniform* velocity (not varying with position):

\(^1\) Actually, a force balance in the $z$ direction demands that the gravitational weight of the wedge be considered, which is proportional to the *volume* of the wedge. However, the pressure forces are proportional to the *areas* of the faces. It can readily be shown that the volume-to-area effect becomes vanishingly small as the wedge becomes infinitesimally small, so that the gravitational weight is inconsequential.
1. If the fluid passes through a plane of area $A$ normal to the direction of the 

 velocity, as shown in Fig. 1.6, the corresponding volumetric flow rate of fluid 

 through the plane is $Q = uA$.

2. The corresponding mass flow rate is $m = \rho Q = \rho uA$, where $\rho$ is the (constant) fluid density. The alternative notation with an overdot, $\dot{m}$, is also used.

3. When velocity is multiplied by mass it gives momentum, a quantity of prime importance in fluid mechanics. The corresponding momentum flow rate passing through the area $A$ is $\dot{M} = \rho u^2 A$.

If $u$ and/or $\rho$ should vary with position, as in Fig. 1.6(b), the corresponding expressions will be seen later to involve integrals over the area $A$: $Q = \int_A u \, dA$, $m = \int_A \rho u \, dA$, $\dot{M} = \int_A \rho u^2 \, dA$.

**Basic laws.** In principle, the laws of fluid mechanics can be stated simply, and—in the absence of relativistic effects—amount to conservation of mass, energy, and momentum. When applying these laws, the procedure is first to identify a system, its boundary, and its surroundings; and second, to identify how the system interacts with its surroundings. Refer to Fig. 1.7 and let the quantity $X$ represent either mass, energy, or momentum. Also recognize that $X$ may be added from the surroundings and transported into the system by an amount $X_{in}$ across the boundary, and may likewise be removed or transported out of the system to the surroundings by an amount $X_{out}$.

![Fig. 1.7 A system and transports to and from it.](image)

The general conservation law gives the increase $\Delta X_{system}$ in the $X$-content of the system as:

$$X_{in} - X_{out} = \Delta X_{system}.$$  \hspace{1cm} (1.4a)

Although this basic law may appear intuitively obvious, it applies only to a very restricted selection of properties $X$. For example, it is not generally true if $X$ is another extensive property such as volume, and is quite meaningless if $X$ is an intensive property such as pressure or temperature.

In certain cases, where $X^i$ is the mass of a definite chemical species $i$, we may also have an amount of creation $X_{created}^i$ or destruction $X_{destroyed}^i$ due to chemical reaction, in which case the general law becomes:

$$X_{in}^i - X_{out}^i + X_{created}^i - X_{destroyed}^i = \Delta X_{system}^i.$$  \hspace{1cm} (1.4b)
Chapter 1—Introduction to Fluid Mechanics

The conservation law will be discussed further in Section 2.1, and is of such fundamental importance that in various guises it will find numerous applications throughout all of this text.

To solve a physical problem, the following information concerning the fluid is also usually needed:

1. The physical properties of the fluid involved, as discussed in Section 1.4.
2. For situations involving fluid flow, a constitutive equation for the fluid, which relates the various stresses to the flow pattern.

1.4 Physical Properties—Density, Viscosity, and Surface Tension

There are three physical properties of fluids that are particularly important: density, viscosity, and surface tension. Each of these will be defined and viewed briefly in terms of molecular concepts, and their dimensions will be examined in terms of mass, length, and time (M, L, and T). The physical properties depend primarily on the particular fluid. For liquids, viscosity also depends strongly on the temperature; for gases, viscosity is approximately proportional to the square root of the absolute temperature. The density of gases depends almost directly on the absolute pressure; for most other cases, the effect of pressure on physical properties can be disregarded.

Typical processes often run almost isothermally, and in these cases the effect of temperature can be ignored. Except in certain special cases, such as the flow of a compressible gas (in which the density is not constant) or a liquid under a very high shear rate (in which viscous dissipation can cause significant internal heating), or situations involving exothermic or endothermic reactions, we shall ignore any variation of physical properties with pressure and temperature.

**Densities of liquids.** Density depends on the mass of an individual molecule and the number of such molecules that occupy a unit of volume. For liquids, density depends primarily on the particular liquid and, to a much smaller extent, on its temperature. Representative densities of liquids are given in Table 1.1.²


**Density.** The density $\rho$ of a fluid is defined as its mass per unit volume, and indicates its inertia or resistance to an accelerating force. Thus:

$$\rho = \frac{\text{mass}}{\text{volume}} = \frac{\text{M}}{\text{L}^3},$$

(1.5)
in which the notation “[=]” is consistently used to indicate the dimensions of a quantity. It is usually understood in Eqn. (1.5) that the volume is chosen so that it is neither so small that it has no chance of containing a representative selection of molecules nor so large that (in the case of gases) changes of pressure cause significant changes of density throughout the volume. A medium characterized by a density is called a continuum, and follows the classical laws of mechanics—including Newton’s law of motion, as described in this book.

<table>
<thead>
<tr>
<th>Liquid</th>
<th>Sp. Gr. s</th>
<th>Density, ρ kg/m³</th>
<th>Density, ρ lbm/ft³</th>
<th>α °C⁻¹</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acetone</td>
<td>0.792</td>
<td>792</td>
<td>49.4</td>
<td>0.00149</td>
</tr>
<tr>
<td>Benzene</td>
<td>0.879</td>
<td>879</td>
<td>54.9</td>
<td>0.00124</td>
</tr>
<tr>
<td>Crude oil, 35°API</td>
<td>0.851</td>
<td>851</td>
<td>53.1</td>
<td>0.00074</td>
</tr>
<tr>
<td>Ethanol</td>
<td>0.789</td>
<td>789</td>
<td>49.3</td>
<td>0.00112</td>
</tr>
<tr>
<td>Glycerol</td>
<td>1.26 (50°C)</td>
<td>1,260</td>
<td>78.7</td>
<td>—</td>
</tr>
<tr>
<td>Kerosene</td>
<td>0.819</td>
<td>819</td>
<td>51.1</td>
<td>0.00093</td>
</tr>
<tr>
<td>Mercury</td>
<td>13.55</td>
<td>13,550</td>
<td>845.9</td>
<td>0.000182</td>
</tr>
<tr>
<td>Methanol</td>
<td>0.792</td>
<td>792</td>
<td>49.4</td>
<td>0.00120</td>
</tr>
<tr>
<td>n-Octane</td>
<td>0.703</td>
<td>703</td>
<td>43.9</td>
<td>—</td>
</tr>
<tr>
<td>n-Pentane</td>
<td>0.630</td>
<td>630</td>
<td>39.3</td>
<td>0.00161</td>
</tr>
<tr>
<td>Water</td>
<td>0.998</td>
<td>998</td>
<td>62.3</td>
<td>0.000207</td>
</tr>
</tbody>
</table>

Degrees A.P.I. (American Petroleum Institute) are related to specific gravity s by the formula:

\[ °A.P.I. = \frac{141.5}{s} - 131.5. \]  

Note that for water, °A.P.I. = 10, with correspondingly higher values for liquids that are less dense. Thus, for the crude oil listed in Table 1.1, Eqn. (1.6) indeed gives 141.5/0.851 – 131.5 = 35 °A.P.I.

**Densities of gases.** For ideal gases, \( pV = nRT \), where \( p \) is the absolute pressure, \( V \) is the volume of the gas, \( n \) is the number of moles (abbreviated as “mol” when used as a unit), \( R \) is the gas constant, and \( T \) is the absolute temperature. If \( M_w \) is the molecular weight of the gas, it follows that:

\[ \rho = \frac{nM_w}{V} = \frac{M_wp}{RT}. \]

---

Thus, the density of an ideal gas depends on the molecular weight, absolute pressure, and absolute temperature. Values of the gas constant $R$ are given in Table 1.2 for various systems of units. Note that degrees Kelvin, formerly represented by “°K,” is now more simply denoted as “K.”

<table>
<thead>
<tr>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.314</td>
<td>J/g-mol K</td>
</tr>
<tr>
<td>0.08314</td>
<td>liter bar/g-mol K</td>
</tr>
<tr>
<td>0.08206</td>
<td>liter atm/g-mol K</td>
</tr>
<tr>
<td>1.987</td>
<td>cal/g-mol K</td>
</tr>
<tr>
<td>10.73</td>
<td>psia ft$^3$/lb-mol °R</td>
</tr>
<tr>
<td>0.7302</td>
<td>ft$^3$ atm/lb-mol °R</td>
</tr>
<tr>
<td>1.545</td>
<td>ft lb$\ell$/lb-mol °R</td>
</tr>
</tbody>
</table>

For a nonideal gas, the compressibility factor $Z$ (a function of $p$ and $T$) is introduced into the denominator of Eqn. (1.7), giving:

$$\rho = \frac{nM_w}{V} = \frac{M_w p}{ZRT}.$$  \hspace{1cm} (1.8)

Thus, the extent to which $Z$ deviates from unity gives a measure of the nonideality of the gas.

The isothermal compressibility of a gas is defined as:

$$\beta = -\frac{1}{V} \left( \frac{\partial V}{\partial p} \right)_T,$$

and equals—at constant temperature—the fractional decrease in volume caused by a unit increase in the pressure. For an ideal gas, $\beta = 1/p$, the reciprocal of the absolute pressure.

The coefficient of thermal expansion $\alpha$ of a material is its isobaric (constant pressure) fractional increase in volume per unit rise in temperature:

$$\alpha = \frac{1}{V} \left( \frac{\partial V}{\partial T} \right)_p.$$  \hspace{1cm} (1.9)

Since, for a given mass, density is inversely proportional to volume, it follows that for moderate temperature ranges (over which $\alpha$ is essentially constant) the density of most liquids is approximately a linear function of temperature:

$$\rho \approx \rho_0[1 - \alpha(T - T_0)],$$  \hspace{1cm} (1.10)
where $\rho_0$ is the density at a reference temperature $T_0$. For an ideal gas, $\alpha = 1/T$, the reciprocal of the absolute temperature.

The specific gravity $s$ of a fluid is the ratio of the density $\rho$ to the density $\rho_{SC}$ of a reference fluid at some standard condition:

$$s = \frac{\rho}{\rho_{SC}}.$$  \hspace{1cm} (1.11)

For liquids, $\rho_{SC}$ is usually the density of water at 4°C, which equals 1,000 g/ml or 1,000 kg/m³. For gases, $\rho_{SC}$ is sometimes taken as the density of air at 60°F and 14.7 psia, which is approximately 0.0759 lbₘ/ft³, and sometimes at 0°C and one atmosphere absolute; since there is no single standard for gases, care must obviously be taken when interpreting published values. For natural gas, consisting primarily of methane and other hydrocarbons, the gas gravity is defined as the ratio of the molecular weight of the gas to that of air (28.8 lbₘ/lb-mol).

Values of the molecular weight $M_w$ are listed in Table 1.3 for several commonly occurring gases, together with their densities at standard conditions of atmospheric pressure and 0°C.

<table>
<thead>
<tr>
<th>Gas</th>
<th>$M_w$</th>
<th>Standard Density</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>kg/m³</td>
<td>lbₘ/ft³</td>
</tr>
<tr>
<td>Air</td>
<td>28.8</td>
<td>1.29</td>
<td>0.0802</td>
</tr>
<tr>
<td>Carbon dioxide</td>
<td>44.0</td>
<td>1.96</td>
<td>0.1225</td>
</tr>
<tr>
<td>Ethylene</td>
<td>28.0</td>
<td>1.25</td>
<td>0.0780</td>
</tr>
<tr>
<td>Hydrogen</td>
<td>2.0</td>
<td>0.089</td>
<td>0.0056</td>
</tr>
<tr>
<td>Methane</td>
<td>16.0</td>
<td>0.714</td>
<td>0.0446</td>
</tr>
<tr>
<td>Nitrogen</td>
<td>28.0</td>
<td>1.25</td>
<td>0.0780</td>
</tr>
<tr>
<td>Oxygen</td>
<td>32.0</td>
<td>1.43</td>
<td>0.0891</td>
</tr>
</tbody>
</table>

**Viscosity.** The viscosity of a fluid measures its resistance to flow under an applied shear stress, as shown in Fig. 1.8(a). There, the fluid is ideally supposed to be confined in a relatively small gap of thickness $h$ between one plate that is stationary and another plate that is moving steadily at a velocity $V$ relative to the first plate.

In practice, the situation would essentially be realized by a fluid occupying the space between two concentric cylinders of large radii rotating relative to each other, as in Fig. 1.1. A steady force $F$ to the right is applied to the upper plate (and, to preserve equilibrium, to the left on the lower plate) in order to maintain a
constant motion and to overcome the viscous friction caused by layers of molecules sliding over one another.

\[ u = \frac{y}{h} V. \]  

(1.12)

Recall that the shear stress \( \tau \) is the tangential applied force \( F \) per unit area:

\[ \tau = \frac{F}{A}, \]  

(1.13)

in which \( A \) is the area of each plate. Experimentally, for a large class of materials, called \textit{Newtonian} fluids, the shear stress is directly proportional to the velocity gradient:

\[ \tau = \mu \frac{du}{dy} = \mu \frac{V}{h}. \]  

(1.14)

The proportionality constant \( \mu \) is called the viscosity of the fluid; its dimensions can be found by substituting those for \( F \) (ML/T^2), \( A \) (L^2), and \( du/dy \) (T^{-1}), giving:

\[ \mu \ [\text{=}] \ \frac{M}{LT}. \]  

(1.15)

Representative units for viscosity are g/cm s (also known as poise, designated by P), kg/m s, and lb_m/ft hr. The centipoise (cP), one hundredth of a poise, is also a convenient unit, since the viscosity of water at room temperature is approximately 0.01 P or 1.0 cP. Table 1.11 gives viscosity conversion factors.

The viscosity of a fluid may be determined by observing the pressure drop when it flows at a known rate in a tube, as analyzed in Section 3.2. More sophisticated
methods for determining the rheological or flow properties of fluids—including viscosity—are also discussed in Chapter 11; such methods often involve containing the fluid in a small gap between two surfaces, moving one of the surfaces, and measuring the force needed to maintain the other surface stationary.

Table 1.4  Viscosity Parameters for Liquids

<table>
<thead>
<tr>
<th>Liquid</th>
<th>a (T in K)</th>
<th>b</th>
<th>a (T in °R)</th>
<th>b</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acetone</td>
<td>14.64</td>
<td>-2.77</td>
<td>16.29</td>
<td>-2.77</td>
</tr>
<tr>
<td>Benzene</td>
<td>21.99</td>
<td>-3.95</td>
<td>24.34</td>
<td>-3.95</td>
</tr>
<tr>
<td>Crude oil, 35° API</td>
<td>53.73</td>
<td>-9.01</td>
<td>59.09</td>
<td>-9.01</td>
</tr>
<tr>
<td>Ethanol</td>
<td>31.63</td>
<td>-5.53</td>
<td>34.93</td>
<td>-5.53</td>
</tr>
<tr>
<td>Glycerol</td>
<td>106.76</td>
<td>-17.60</td>
<td>117.22</td>
<td>-17.60</td>
</tr>
<tr>
<td>Kerosene</td>
<td>33.41</td>
<td>-5.72</td>
<td>36.82</td>
<td>-5.72</td>
</tr>
<tr>
<td>Methanol</td>
<td>22.18</td>
<td>-3.99</td>
<td>24.56</td>
<td>-3.99</td>
</tr>
<tr>
<td>Octane</td>
<td>17.86</td>
<td>-3.25</td>
<td>19.80</td>
<td>-3.25</td>
</tr>
<tr>
<td>Pentane</td>
<td>13.46</td>
<td>-2.62</td>
<td>15.02</td>
<td>-2.62</td>
</tr>
<tr>
<td>Water</td>
<td>29.76</td>
<td>-5.24</td>
<td>32.88</td>
<td>-5.24</td>
</tr>
</tbody>
</table>

The kinematic viscosity $\nu$ is the ratio of the viscosity to the density:

$$\nu = \frac{\mu}{\rho}, \quad (1.16)$$

and is important in cases in which significant viscous and gravitational forces coexist. The reader can check that the dimensions of $\nu$ are $L^2/T$, which are identical to those for the diffusion coefficient $D$ in mass transfer and for the thermal diffusivity $\alpha = k/\rho c_p$ in heat transfer. There is a definite analogy among the three quantities—indeed, as seen later, the value of the kinematic viscosity governs the rate of “diffusion” of momentum in the laminar and turbulent flow of fluids.

Viscosities of liquids. The viscosities $\mu$ of liquids generally vary approximately with absolute temperature $T$ according to:

$$\ln \mu \doteq a + b \ln T \quad \text{or} \quad \mu \doteq e^{a+b\ln T}, \quad (1.17)$$

and—to a good approximation—are independent of pressure. Assuming that $\mu$ is measured in centipoise and that $T$ is either in degrees Kelvin or Rankine, appropriate parameters $a$ and $b$ are given in Table 1.4 for several representative liquids. The resulting values for viscosity are approximate, suitable for a first design only.
Viscosities of gases. The viscosity $\mu$ of many gases is approximated by the formula:

$$\mu \approx \mu_0 \left( \frac{T}{T_0} \right)^n,$$

in which $T$ is the absolute temperature (Kelvin or Rankine), $\mu_0$ is the viscosity at an absolute reference temperature $T_0$, and $n$ is an empirical exponent that best fits the experimental data. The values of the parameters $\mu_0$ and $n$ for atmospheric pressure are given in Table 1.5; recall that to a first approximation, the viscosity of a gas is independent of pressure. The values $\mu_0$ are given in centipoise and correspond to a reference temperature of $T_0 = 273$ K $= 492$ °R.

<table>
<thead>
<tr>
<th>Gas</th>
<th>$\mu_0$, cP</th>
<th>$n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air</td>
<td>0.0171</td>
<td>0.768</td>
</tr>
<tr>
<td>Carbon dioxide</td>
<td>0.0137</td>
<td>0.935</td>
</tr>
<tr>
<td>Ethylene</td>
<td>0.0096</td>
<td>0.812</td>
</tr>
<tr>
<td>Hydrogen</td>
<td>0.0084</td>
<td>0.695</td>
</tr>
<tr>
<td>Methane</td>
<td>0.0120</td>
<td>0.873</td>
</tr>
<tr>
<td>Nitrogen</td>
<td>0.0166</td>
<td>0.756</td>
</tr>
<tr>
<td>Oxygen</td>
<td>0.0187</td>
<td>0.814</td>
</tr>
</tbody>
</table>

Surface tension. Surface tension is the tendency of the surface of a liquid to behave like a stretched elastic membrane. There is a natural tendency for liquids to minimize their surface area. The obvious case is that of a liquid droplet on a horizontal surface that is not wetted by the liquid—mercury on glass, or water on a surface that also has a thin oil film on it. For small droplets, such as those on the left of Fig. 1.9, the droplet adopts a shape that is almost perfectly spherical, because in this configuration there is the least surface area for a given volume.

![Fig. 1.9](image)

*Fig. 1.9 The larger droplets are flatter because gravity is becoming more important than surface tension.*

---

4 We recommend that this subsection be omitted at a first reading, because the concept of surface tension is somewhat involved and is relevant only to a small part of this book.
For larger droplets, the shape becomes somewhat flatter because of the increasingly important gravitational effect, which is roughly proportional to $a^3$, where $a$ is the approximate droplet radius, whereas the surface area is proportional only to $a^2$. Thus, the ratio of gravitational to surface tension effects depends roughly on the value of $a^3/a^2 = a$, and is therefore increasingly important for the larger droplets, as shown to the right in Fig. 1.9. Overall, the situation is very similar to that of a water-filled balloon, in which the water accounts for the gravitational effect and the balloon acts like the surface tension.

A fundamental property is the surface energy, which is defined with reference to Fig. 1.10(a). A molecule I, situated in the interior of the liquid, is attracted equally in all directions by its neighbors. However, a molecule S, situated in the surface, experiences a net attractive force into the bulk of the liquid. (The vapor above the surface, being comparatively rarefied, exerts a negligible force on molecule S.) Therefore, work has to be done against such a force in bringing an interior molecule to the surface. Hence, an energy $\sigma$, called the surface energy, can be attributed to a unit area of the surface.

An equivalent viewpoint is to consider the surface tension $T$ existing per unit distance of a line drawn in the surface, as shown in Fig. 1.10(b). Suppose that such a tension has moved a distance $L$, thereby creating an area $WL$ of fresh surface. The work done is the product of the force, $TW$, and the distance $L$ through which it moves, namely $TWL$, and this must equal the newly acquired surface energy $\sigma WL$. Therefore, $T = \sigma$; both quantities have units of force per unit distance, such as N/m, which is equivalent to energy per unit area, such as J/m$^2$.

We next find the amount $p_1 - p_2$ by which the pressure $p_1$ inside a liquid droplet of radius $r$, shown in Fig. 1.11(a), exceeds the pressure $p_2$ of the surrounding vapor. Fig. 1.11(b) illustrates the equilibrium of the upper hemisphere of the droplet, which is also surrounded by an imaginary cylindrical “control surface” ABCD, on which forces in the vertical direction will soon be equated. Observe that the
internal pressure $p_1$ is trying to blow apart the two hemispheres (the lower one is not shown), whereas the surface tension $\sigma$ is trying to pull them together.

![Diagram](image)

(a) Liquid droplet  
(b) Forces in equilibrium

**Fig. 1.11 Pressure change across a curved surface.**

In more detail, there are two different types of forces to be considered:

1. That due to the *pressure difference* between the pressure inside the droplet and the vapor outside, each acting on an area $\pi r^2$ (that of the circles CD and AB):
   \[
   (p_1 - p_2)\pi r^2. \tag{1.19}
   \]

2. That due to surface tension, which acts on the circumference of length $2\pi r$:
   \[
   2\pi r\sigma. \tag{1.20}
   \]

At equilibrium, these two forces are equated, giving:

\[
\Delta p = p_1 - p_2 = \frac{2\sigma}{r}. \tag{1.21}
\]

That is, there is a *higher* pressure on the concave or droplet side of the interface. What would the pressure change be for a *bubble* instead of a droplet? Why?

More generally, if an interface has principal radii of curvature $r_1$ and $r_2$, the increase in pressure can be shown to be:

\[
p_1 - p_2 = \sigma \left( \frac{1}{r_1} + \frac{1}{r_2} \right). \tag{1.22}
\]

For a sphere of radius $r$, as in Fig. 1.11, both radii are equal, so that $r_1 = r_2 = r$, and $p_1 - p_2 = 2\sigma/r$. Problem 1.31 involves a situation in which $r_1 \neq r_2$. The radii $r_1$ and $r_2$ will have the same sign if the corresponding centers of curvature are on the same side of the interface; if not, they will be of opposite sign. Appendix A contains further information about the curvature of a surface.
A brief description of simple experiments for measuring the surface tension $\sigma$ of a liquid, shown in Fig. 1.12, now follows:

(a) In the capillary-rise method, a narrow tube of internal radius $a$ is dipped vertically into a pool of liquid, which then rises to a height $h$ inside the tube; if the contact angle (the angle between the free surface and the wall) is $\theta$, the meniscus will be approximated by part of the surface of a sphere; from the geometry shown in the enlargement on the right-hand side of Fig. 1.12(a) the radius of the sphere is seen to be $r = a / \cos \theta$. Since the surface is now concave on the air side, the reverse of Eqn. (1.21) occurs, and $p_2 = p_1 - 2\sigma / r$, so that $p_2$ is below atmospheric pressure $p_1$. Now follow the path 1–2–3–4, and observe that $p_4 = p_3$ because points
3 and 4 are at the same elevation in the same liquid. Thus, the pressure at point 4 is:

\[ p_4 = p_1 - \frac{2\sigma}{r} + \rho gh. \]

However, \( p_4 = p_1 \) since both of these are at atmospheric pressure. Hence, the surface tension is given by the relation:

\[ \sigma = \frac{1}{2} \rho gh r = \frac{\rho gh a}{2 \cos \theta}. \] (1.23)

In many cases—for complete wetting of the surface—\( \theta \) is essentially zero and \( \cos \theta = 1 \). However, for liquids such as mercury in glass, there may be a complete non-wetting of the surface, in which case \( \theta = \pi \), so that \( \cos \theta = -1 \); the result is that the liquid level in the capillary is then depressed below that in the surrounding pool.

(b) In the drop-weight method, a liquid droplet is allowed to form very slowly at the tip of a capillary tube of outer diameter \( D \). The droplet will eventually grow to a size where its weight just overcomes the surface-tension force \( \pi D \sigma \) holding it up. At this stage, it will detach from the tube, and its weight \( w = Mg \) can be determined by catching it in a small pan and weighing it. By equating the two forces, the surface tension is then calculated from:

\[ \sigma = \frac{w}{\pi D}. \] (1.24)

(c) In the ring tensiometer, a thin wire ring, suspended from the arm of a sensitive balance, is dipped into the liquid and gently raised, so that it brings a thin liquid film up with it. The force \( F \) needed to support the film is measured by the balance. The downward force exerted on a unit length of the ring by one side of the film is the surface tension; since there are two sides to the film, the total force is \( 2P\sigma \), where \( P \) is the circumference of the ring. The surface tension is therefore determined as:

\[ \sigma = \frac{F}{2P}. \] (1.25)

In common with most experimental techniques, all three methods described above require slight modifications to the results expressed in Eqns. (1.23)–(1.25) because of imperfections in the simple theories.

Surface tension generally appears only in situations involving either free surfaces (liquid/gas or liquid/solid boundaries) or interfaces (liquid/liquid boundaries); in the latter case, it is usually called the interfacial tension.

Representative values for the surface tensions of liquids at 20°C, in contact either with air or their vapor (there is usually little difference between the two), are given in Table 1.6.\(^5\)

---

\(^5\) The values for surface tension have been obtained from the *CRC Handbook of Chemistry and Physics*, 48th ed., The Chemical Rubber Co., Cleveland, OH, 1967.
Table 1.6  Surface Tensions

<table>
<thead>
<tr>
<th>Liquid</th>
<th>σ</th>
<th>dynes/cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acetone</td>
<td>23.70</td>
<td></td>
</tr>
<tr>
<td>Benzene</td>
<td>28.85</td>
<td></td>
</tr>
<tr>
<td>Ethanol</td>
<td>22.75</td>
<td></td>
</tr>
<tr>
<td>Glycerol</td>
<td>63.40</td>
<td></td>
</tr>
<tr>
<td>Mercury</td>
<td>435.5</td>
<td></td>
</tr>
<tr>
<td>Methanol</td>
<td>22.61</td>
<td></td>
</tr>
<tr>
<td>n-Octane</td>
<td>21.80</td>
<td></td>
</tr>
<tr>
<td>Water</td>
<td>72.75</td>
<td></td>
</tr>
</tbody>
</table>

1.5 Units and Systems of Units

**Mass, weight, and force.** The mass $M$ of an object is a measure of the amount of matter it contains and will be constant, since it depends on the number of constituent molecules and their masses. On the other hand, the weight $w$ of the object is the gravitational force on it, and is equal to $Mg$, where $g$ is the local gravitational acceleration. Mostly, we shall be discussing phenomena occurring at the surface of the earth, where $g$ is approximately $32.174 \text{ ft/s}^2 = 9.807 \text{ m/s}^2 = 980.7 \text{ cm/s}^2$. For much of this book, these values are simply taken as 32.2, 9.81, and 981, respectively.

Table 1.7  Representative Units of Force

<table>
<thead>
<tr>
<th>System</th>
<th>Units of Force</th>
<th>Customary Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>SI</td>
<td>kg m/s$^2$</td>
<td>newton</td>
</tr>
<tr>
<td>CGS</td>
<td>g cm/s$^2$</td>
<td>dyne</td>
</tr>
<tr>
<td>FPS</td>
<td>lb$_m$ ft/s$^2$</td>
<td>poundal</td>
</tr>
</tbody>
</table>

Newton’s second law of motion states that a force $F$ applied to a mass $M$ will give it an acceleration $a$:

$$F = Ma,$$  (1.26)

from which is apparent that force has dimensions $\text{ML/T}^2$. Table 1.7 gives the corresponding units of force in the SI (meter/kilogram/second), CGS (centimeter/gram/second), and FPS (foot/pound/second) systems.
The poundal is now an archaic unit, hardly ever used. Instead, the **pound force**, \( \text{lb}_f \), is much more common in the English system; it is defined as the gravitational force on 1 lb\(_m\), which, if left to fall freely, will do so with an acceleration of 32.2 \( \text{ft/s}^2 \). Hence:

\[
1 \text{ lb}_f = 32.2 \frac{\text{lb}_m \text{ ft}}{\text{s}^2} = 32.2 \text{ poundals.} \tag{1.27}
\]

<table>
<thead>
<tr>
<th>Physical Quantity</th>
<th>Name of Unit</th>
<th>Symbol for Unit</th>
<th>Definition of Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Basic Units</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Length</td>
<td>meter</td>
<td>m</td>
<td>–</td>
</tr>
<tr>
<td>Mass</td>
<td>kilogram</td>
<td>kg</td>
<td>–</td>
</tr>
<tr>
<td>Time</td>
<td>second</td>
<td>s</td>
<td>–</td>
</tr>
<tr>
<td>Temperature</td>
<td>degree</td>
<td></td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>Kelvin</td>
<td>K</td>
<td>–</td>
</tr>
<tr>
<td><strong>Supplementary Unit</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Plane angle</td>
<td>radian</td>
<td>rad</td>
<td>–</td>
</tr>
<tr>
<td><strong>Derived Units</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Acceleration</td>
<td></td>
<td>m/s(^2)</td>
<td></td>
</tr>
<tr>
<td>Angular velocity</td>
<td></td>
<td>rad/s</td>
<td></td>
</tr>
<tr>
<td>Density</td>
<td></td>
<td>kg/m(^3)</td>
<td></td>
</tr>
<tr>
<td>Energy</td>
<td>joule</td>
<td>J</td>
<td>kg m(^2)/s(^2)</td>
</tr>
<tr>
<td>Force</td>
<td>newton</td>
<td>N</td>
<td>kg m/s(^2)</td>
</tr>
<tr>
<td>Kinematic viscosity</td>
<td></td>
<td>m(^2)/s</td>
<td></td>
</tr>
<tr>
<td>Power</td>
<td>watt</td>
<td>W</td>
<td>kg m(^2)/s(^3) (J/s)</td>
</tr>
<tr>
<td>Pressure</td>
<td>pascal</td>
<td>Pa</td>
<td>kg/m s(^2) (N/m(^2))</td>
</tr>
<tr>
<td>Velocity</td>
<td></td>
<td>m/s</td>
<td></td>
</tr>
<tr>
<td>Viscosity</td>
<td></td>
<td>kg/m s</td>
<td></td>
</tr>
</tbody>
</table>

When using \( \text{lb}_f \) in the \( \text{ft}, \text{lb}_m, \text{s} \) (FPS) system, the following conversion factor, commonly called “\( g_c \),” will almost invariably be needed:

\[
g_c = 32.2 \frac{\text{lb}_m \text{ ft/s}^2}{\text{lb}_f} = 32.2 \frac{\text{lb}_m \text{ ft}}{\text{lb}_f \text{ s}^2}. \tag{1.28}
\]
Some writers incorporate $g_c$ into their equations, but this approach may be confusing since it virtually implies that one particular set of units is being used, and hence tends to rob the equations of their generality. Why not, for example, also incorporate the conversion factor of 144 in$^2$/ft$^2$ into equations where pressure is expressed in lb/in$^2$? We prefer to omit all conversion factors in equations, and introduce them only as needed in evaluating expressions numerically. If the reader is in any doubt, units should always be checked when performing calculations.

**SI Units.** The most systematically developed and universally accepted set of units occurs in the *SI* units or *Système International d’Unités*; the subset we mainly need is shown in Table 1.8.

The basic units are again the meter, kilogram, and second (m, kg, and s); from these, certain *derived* units can also be obtained. Force (kg m/s$^2$) has already been discussed; energy is the product of force and length; power amounts to energy per unit time; surface tension is energy per unit area or force per unit length, and so on. Some of the units have names, and these, together with their abbreviations, are also given in Table 1.8.

<table>
<thead>
<tr>
<th>Physical Quantity</th>
<th>Name of Unit</th>
<th>Symbol for Unit</th>
<th>Definition of Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area</td>
<td>hectare</td>
<td>ha</td>
<td>$10^4$ m$^2$</td>
</tr>
<tr>
<td>Kinematic viscosity</td>
<td>stokes</td>
<td>St</td>
<td>$10^{-4}$ m$^2$/s</td>
</tr>
<tr>
<td>Length</td>
<td>micron</td>
<td>μm</td>
<td>$10^{-6}$ m</td>
</tr>
<tr>
<td>Mass</td>
<td>tonne</td>
<td>t</td>
<td>$10^3$ kg = Mg</td>
</tr>
<tr>
<td>Pressure</td>
<td>bar</td>
<td>bar</td>
<td>$10^5$ N/m$^2$</td>
</tr>
<tr>
<td>Viscosity</td>
<td>poise</td>
<td>P</td>
<td>$10^{-1}$ kg/m s</td>
</tr>
<tr>
<td>Volume</td>
<td>liter</td>
<td>l</td>
<td>$10^{-3}$ m$^3$</td>
</tr>
</tbody>
</table>

Tradition dies hard, and certain other “metric” units are so well established that they may be used as *auxiliary* units; these are shown in Table 1.9. The *gram* is the classic example. Note that the basic SI unit of mass (kg) is even represented in terms of the gram, and has not yet been given a name of its own!

Table 1.10 shows some of the acceptable prefixes that can be used for accommodating both small and large quantities. For example, to avoid an excessive number of decimal places, 0.000001 s is normally better expressed as 1 μs (one microsecond). Note also, for example, that 1 μkg should be written as 1 mg—one prefix being better than two.

---

6 For an excellent discussion, on which Tables 1.8 and 1.9 are based, see *Metrication in Scientific Journals*, published by The Royal Society, London, 1968.
Chapter 1—Introduction to Fluid Mechanics

Table 1.10 Prefixes for Fractions and Multiples

<table>
<thead>
<tr>
<th>Factor</th>
<th>Name</th>
<th>Symbol</th>
<th>Factor</th>
<th>Name</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{-12}$</td>
<td>pico</td>
<td>p</td>
<td>$10^3$</td>
<td>kilo</td>
<td>k</td>
</tr>
<tr>
<td>$10^{-9}$</td>
<td>nano</td>
<td>n</td>
<td>$10^6$</td>
<td>mega</td>
<td>M</td>
</tr>
<tr>
<td>$10^{-6}$</td>
<td>micro</td>
<td>μ</td>
<td>$10^9$</td>
<td>giga</td>
<td>G</td>
</tr>
<tr>
<td>$10^{-3}$</td>
<td>milli</td>
<td>m</td>
<td>$10^{12}$</td>
<td>tera</td>
<td>T</td>
</tr>
</tbody>
</table>

Some of the more frequently used conversion factors are given in Table 1.11.

Example 1.1—Units Conversion

Part 1. Express 65 mph in (a) ft/s, and (b) m/s.

Solution

The solution is obtained by employing conversion factors taken from Table 1.11:

(a) $65 \frac{\text{mile}}{\text{hr}} \times \frac{1}{3,600} \frac{\text{hr}}{\text{s}} \times 5,280 \frac{\text{ft}}{\text{mile}} = 95.33 \frac{\text{ft}}{\text{s}}$.

(b) $95.33 \frac{\text{ft}}{\text{s}} \times 0.3048 \frac{\text{m}}{\text{ft}} = 29.06 \frac{\text{m}}{\text{s}}$.

Part 2. The density of 35°API crude oil is 53.1 lbm/ft³ at 68°F and its viscosity is 32.8 lbm/ft/hr. What are its density, viscosity, and kinematic viscosity in SI units?

Solution

\[
\rho = 53.1 \frac{\text{lbm}}{\text{ft}^3} \times 0.4536 \frac{\text{kg}}{\text{lbm}} \times \frac{1}{0.3048^3} \frac{\text{m}^3}{\text{ft}^3} = 851 \frac{\text{kg}}{\text{m}^3}.
\]

\[
\mu = 32.8 \frac{\text{lbm}}{\text{ft} \text{hr}} \times \frac{1}{2.419} \frac{\text{centipoise}}{\text{lbm/ft hr}} \times 0.01 \frac{\text{poise}}{\text{centipoise}} = 0.136 \text{ poise}.
\]

Or, converting to SI units, noting that P is the symbol for poise, and evaluating $\nu$:

\[
\mu = 0.136 P \times 0.1 \frac{\text{kg/m s}}{P} = 0.0136 \frac{\text{kg}}{\text{m s}}.
\]

\[
\nu = \frac{\mu}{\rho} = \frac{0.0136 \text{ kg/m s}}{851 \text{ kg/m}^3} = 1.60 \times 10^{-5} \frac{\text{m}^2}{\text{s}} (= 0.160 \text{ St}).
\]
Table 1.11 Commonly Used Conversion Factors

<table>
<thead>
<tr>
<th>Category</th>
<th>Conversion Factor</th>
<th>Equivalent</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Area</strong></td>
<td>1 mile(^2) = 640 acres</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1 acre = 0.4047 ha</td>
<td></td>
</tr>
<tr>
<td><strong>Energy</strong></td>
<td>1 BTU = 1,055 J</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1 cal = 4.184 J</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1 J = 0.7376 ft lb(_f)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1 erg = 1 dyne cm</td>
<td></td>
</tr>
<tr>
<td><strong>Force</strong></td>
<td>1 lb(_f) = 4.448 N</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1 N = 0.2248 lb(_f)</td>
<td></td>
</tr>
<tr>
<td><strong>Length</strong></td>
<td>1 ft = 0.3048 m</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1 m = 3.281 ft</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1 mile = 5,280 ft</td>
<td></td>
</tr>
<tr>
<td><strong>Mass</strong></td>
<td>1 lb(_m) = 0.4536 kg</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1 kg = 2.205 lb(_m)</td>
<td></td>
</tr>
<tr>
<td><strong>Power</strong></td>
<td>1 HP = 550 ft lb(_f)/s</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1 kW = 737.6 ft lb(_f)/s</td>
<td></td>
</tr>
<tr>
<td><strong>Pressure</strong></td>
<td>1 atm = 14.696 lb(_f)/in(^2)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1 atm = 1.0133 bar</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1 atm = 1.0133 × 10(^5) Pa</td>
<td></td>
</tr>
<tr>
<td><strong>Time</strong></td>
<td>1 day = 24 hr</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1 hr = 60 min</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1 min = 60 s</td>
<td></td>
</tr>
<tr>
<td><strong>Viscosity</strong></td>
<td>1 cP = 2.419 lb(_m)/ft hr</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1 cP = 0.001 kg/m s</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1 cP = 0.000672 lb(_m)/ft s</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1 lb(_f) s/ft(^2) = 4.788 × 10(^4) cP</td>
<td></td>
</tr>
<tr>
<td><strong>Volume</strong></td>
<td>1 ft(^3) = 7.481 U.S. gal</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1 U.S. gal = 3.785 l</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1 m(^3) = 264.2 U.S. gal</td>
<td></td>
</tr>
</tbody>
</table>

Example 1.2—Mass of Air in a Room

Estimate the mass of air in your classroom, which is 80 ft wide, 40 ft deep, and 12 ft high. The gas constant is \(R = 10.73\) psia ft\(^3\)/lb-mol °R.

**Solution**

The volume of the classroom, shown in Fig. E1.2, is:

\[ V = 80 \times 40 \times 12 = 3.84 \times 10^4 \text{ ft}^3. \]
Chapter 1—Introduction to Fluid Mechanics

If the air is approximately 20% oxygen and 80% nitrogen, its mean molecular weight is \( M_w = 0.8 \times 28 + 0.2 \times 32 = 28.8 \text{ lb}_m/\text{lb-mol} \). From the gas law, assuming an absolute pressure of \( p = 14.7 \text{ psia} \) and a temperature of \( 70^\circ F = 530^\circ R \), the density is:

\[
\rho = \frac{M_w p}{R T} = \frac{28.8 \text{ (lb}_m/\text{lb mol)} \times 14.7 \text{ (psia)}}{10.73 \text{ (psia ft}^3/\text{lb mol }^\circ R) \times 530 \text{ (}^\circ R)} = 0.0744 \text{ lb}_m/\text{ft}^3.
\]

Hence, the mass of air is:

\[
M = \rho V = 0.0744 \text{ (lb}_m/\text{ft}^3) \times 3.84 \times 10^4 \text{ (ft}^3) = 2,860 \text{ lb}_m.
\]

For the rest of the book, manipulation of units will often be less detailed; the reader should always check if there is any doubt.

1.6 Hydrostatics

**Variation of pressure with elevation.** Here, we investigate how the pressure in a stationary fluid varies with elevation \( z \). The result is useful because it can answer questions such as “What is the pressure at the summit of Mt. Annapurna?” or “What forces are exerted on the walls of an oil storage tank?” Consider a hypothetical differential cylindrical element of fluid of cross-sectional area \( A \), height \( dz \), and volume \( A dz \), which is also surrounded by the same fluid, as shown in Fig. 1.13. Its weight, being the downwards gravitational force on its mass, is \( dW = \rho A dz \, g \).

Two completely equivalent approaches will be presented:

**Method 1.** Let \( p \) denote the pressure at the base of the cylinder; since \( p \) changes at a rate \( dp/dz \) with elevation, the pressure is found either from Taylor’s expansion or the definition of a derivative to be \( p + (dp/dz)dz \) at the top of the cylinder.\(^7\) (Note that we do not anticipate a reduction of pressure with elevation here; hence, the plus sign is used. If, indeed—as proves to be the case—pressure falls with increasing elevation, then the subsequent development will tell us that

---

\(^7\) Further details of this fundamental statement can be found in Appendix A and must be fully understood, because similar assertions appear repeatedly throughout the book.
$dp/dz$ is negative.) Hence, the fluid exerts an *upward* force of $pA$ on the base of the cylinder, and a *downward* force of $[p + (dp/dz)dz]A$ on the top of the cylinder.

Next, apply Newton’s second law of motion by equating the net upward force to the mass times the acceleration—which is zero, since the cylinder is stationary:

$$pA - \left(p + \frac{dp}{dz}dz\right)A - \rho Adz g = (\rho Adz) \times 0 = 0.$$  \hspace{1cm} (1.29)

Cancellation of $pA$ and division by $Adz$ leads to the following *differential equation*, which governs the rate of change of pressure with elevation:

$$\frac{dp}{dz} = -\rho g.$$  \hspace{1cm} (1.30)

*Fig. 1.13  Forces acting on a cylinder of fluid.*

**Method 2.** Let $p_z$ and $p_{z+dz}$ denote the pressures at the base and top of the cylinder, where the elevations are $z$ and $z+dz$, respectively. Hence, the fluid exerts an *upward* force of $p_z A$ on the base of the cylinder, and a *downward* force of $p_{z+dz} A$ on the top of the cylinder. Application of Newton’s second law of motion gives:

$$p_zA - p_{z+dz}A - \rho Adz g = (\rho Adz) \times 0 = 0.$$  \hspace{1cm} (1.31)

Isolation of the two pressure terms on the left-hand side and division by $Adz$ gives:

$$\frac{p_{z+dz} - p_z}{dz} = -\rho g.$$ \hspace{1cm} (1.32)

As $dz$ tends to zero, the left-hand side of Eqn. (1.32) becomes the derivative $dp/dz$, leading to the same result as previously:

$$\frac{dp}{dz} = -\rho g.$$ \hspace{1cm} (1.30)

The same conclusion can also be obtained by considering a cylinder of *finite* height $\Delta z$ and then letting $\Delta z$ approach zero.
Note that Eqn. (1.30) predicts a pressure decrease in the vertically upward direction at a rate that is proportional to the local density. Such pressure variations can readily be detected by the ear when traveling quickly in an elevator in a tall building, or when taking off in an airplane. The reader must thoroughly understand both the above approaches. For most of this book, we shall use Method 1, because it eliminates the steps of taking the limit of \( dz \to 0 \) and invoking the definition of the derivative.

**Pressure in a liquid with a free surface.** In Fig. 1.14, the pressure is \( p_s \) at the free surface, and we wish to find the pressure \( p \) at a depth \( H \) below the free surface—of water in a swimming pool, for example.

\[ p = p_s + \rho g H, \]  
\[ (1.34) \]

which predicts a linear increase of pressure with distance downward from the free surface. For large depths, such as those encountered by deep-sea divers, very substantial pressures will result.
Example 1.3—Pressure in an Oil Storage Tank

What is the absolute pressure at the bottom of the cylindrical tank of Fig. E1.3, filled to a depth of $H$ with crude oil, with its free surface exposed to the atmosphere? The specific gravity of the crude oil is 0.846. Give the answers for (a) $H = 15.0$ ft (pressure in lb$_f$/in$^2$), and (b) $H = 5.0$ m (pressure in Pa and bar). What is the purpose of the surrounding dike?

![Crude oil storage tank](image)

Fig. E1.3 Crude oil storage tank.

Solution

(a) The pressure is that of the atmosphere, $p_a$, plus the increase due to a column of depth $H = 15.0$ ft. Thus, setting $p_s = p_a$, Eqn. (1.34) gives:

$$p = p_a + \rho g H$$

$$= 14.7 + \frac{0.846 \times 62.3 \times 32.2 \times 15.0}{144 \times 32.2}$$

$$= 14.7 + 5.49 = 20.2\text{ psi}.$$

The reader should check the units, noting that the 32.2 in the numerator is $g = \text{ft/s}^2$, and that the 32.2 in the denominator is $g_c = \text{lb}_m\text{ ft/lb}_f\text{ s}^2$.

(b) For SI units, no conversion factors are needed. Noting that the density of water is 1,000 kg/m$^3$, and that $p_a = 1.01 \times 10^5$ Pa absolute:

$$p = 1.01 \times 10^5 + 0.846 \times 1,000 \times 9.81 \times 5.0 = 1.42 \times 10^5 \text{ Pa} = 1.42\text{ bar}.$$

In the event of a tank rupture, the dike contains the leaking oil and facilitates prevention of spreading fire and contamination of the environment.

Epilogue

When he arrived at work in an oil refinery one morning, the author saw first-hand the consequences of an inadequately vented oil-storage tank. Rain during the night had caused partial condensation of vapor inside the tank, whose pressure had become sufficiently lowered so that the external atmospheric pressure had crumpled the steel tank just as if it were a flimsy tin can. The refinery manager was not pleased.
Example 1.4—Multiple Fluid Hydrostatics

The U-tube shown in Fig. E1.4 contains oil and water columns, between which there is a long trapped air bubble. For the indicated heights of the columns, find the specific gravity of the oil.

**Figure E1.4** Oil/air/water system.

**Solution**

The pressure $p_2$ at point 2 may be deduced by starting with the pressure $p_1$ at point 1 and adding or subtracting, as appropriate, the hydrostatic pressure changes due to the various columns of fluid. Note that the width of the U-tube (2.0 ft) is irrelevant, since there is no change in pressure in the horizontal leg. We obtain:

$$p_2 = p_1 + \rho_o gh_1 + \rho_a gh_2 + \rho_w gh_3 - \rho_w gh_4,$$

(E1.4.1)

in which $\rho_o$, $\rho_a$, and $\rho_w$ denote the densities of oil, air, and water, respectively. Since the density of the air is very small compared to that of oil or water, the term containing $\rho_a$ can be neglected. Also, $p_1 = p_2$, because both are equal to atmospheric pressure. Equation (E1.4.1) can then be solved for the specific gravity $s_o$ of the oil:

$$s_o = \frac{\rho_o}{\rho_w} = \frac{h_4 - h_3}{h_1} = \frac{3.0 - 1.0}{2.5} = 0.80.$$ 

Pressure variations in a gas. For a gas, the density is no longer constant, but is a function of pressure (and of temperature—although temperature variations are usually less significant than those of pressure), and there are two approaches:

1. For *small* changes in elevation, the assumption of constant density can still be made, and equations similar to Eqn. (1.34) are still approximately valid.
2. For moderate or large changes in elevation, the density in Eqn. (1.30) is given by Eqn. (1.7) or (1.8), $\rho = M_w p / RT$ or $\rho = M_w p / ZRT$, depending on whether
the gas is ideal or nonideal. It is understood that absolute pressure and temperature must always be used whenever the gas law is involved. A separation of variables can still be made, followed by integration, but the result will now be more complicated because the term $dp/p$ occurs, leading—at the simplest (for an isothermal situation)—to a decreasing exponential variation of pressure with elevation.

**Example 1.5—Pressure Variations in a Gas**

For a gas of molecular weight $M_w$ (such as the earth’s atmosphere), investigate how the pressure $p$ varies with elevation $z$ if $p = p_0$ at $z = 0$. Assume that the temperature $T$ is constant. What approximation may be made for small elevation increases? Explain how you would proceed for the nonisothermal case, in which $T = T(z)$ is a known function of elevation.

**Solution**

Assuming ideal gas behavior, Eqns. (1.30) and (1.7) give:

$$\frac{dp}{dz} = -\rho g = -\frac{M_w p}{RT} g.$$  \hspace{1cm} (E1.5.1)

Separation of variables and integration between appropriate limits yields:

$$\int_{p_0}^{p} \frac{dp}{p} = \ln \frac{p}{p_0} = -\int_0^z \frac{M_w g}{RT} dz = -\frac{M_w g}{RT} \int_0^z dz = -\frac{M_w g z}{RT},$$  \hspace{1cm} (E1.5.2)

since $M_w g/RT$ is constant. Hence, there is an exponential decrease of pressure with elevation, as shown in Fig. E1.5:

$$p = p_0 \exp \left( -\frac{M_w g}{RT} z \right).$$  \hspace{1cm} (E1.5.3)

Since a Taylor’s expansion gives $e^{-x} = 1 - x + x^2/2 - \ldots$, the pressure is approximated by:

$$p \approx p_0 \left[ 1 - \frac{M_w g}{RT} z + \left( \frac{M_w g}{RT} \right)^2 \frac{z^2}{2} \right].$$  \hspace{1cm} (E1.5.4)

For small values of $M_w g z/RT$, the last term is an insignificant second-order effect (compressibility effects are unimportant), and we obtain:

$$p \approx p_0 - \frac{M_w p_0}{RT} g z = p_0 - \rho_0 g z,$$  \hspace{1cm} (E1.5.5)

in which $\rho_0$ is the density at elevation $z = 0$; this approximation—essentially one of constant density—is shown as the dashed line in Fig. E1.5 and is clearly
applicable only for a small change of elevation. Problem 1.19 investigates the upper limit on \( z \) for which this linear approximation is realistic. If there are significant elevation changes—as in Problems 1.16 and 1.30—the approximation of Eqn. (E1.5.5) cannot be used with any accuracy. Observe with caution that the Taylor’s expansion is only a vehicle for demonstrating what happens for small values of \( M_w g z/RT \). Actual calculations for larger values of \( M_w g z/RT \) should be made using Eqn. (E1.5.3), not Eqn. (E1.5.4).

For the case in which the temperature is not constant, but is a known function \( T(z) \) of elevation (as might be deduced from observations made by a meteorological balloon), it must be included inside the integral:

\[
\int_{p_1}^{p_2} \frac{dp}{p} = -\frac{M_w g}{R} \int_0^z \frac{dz}{T(z)}.
\]

Since \( T(z) \) is unlikely to be a simple function of \( z \), a numerical method—such as Simpson’s rule in Appendix A—will probably have to be used to approximate the second integral of Eqn. (E1.5.6).

**Total force on a dam or lock gate.** Fig. 1.15 shows the side and end elevations of a dam or lock gate of depth \( D \) and width \( W \). An expression is needed for the total horizontal force \( F \) exerted by the liquid on the dam, so that the latter can be made of appropriate strength. Similar results would apply for liquids in storage tanks. Gauge pressures are used for simplicity, with \( p = 0 \) at the free surface and in the air outside the dam. Absolute pressures could also be employed, but would merely add a constant atmospheric pressure everywhere, and would eventually be canceled out. If the coordinate \( z \) is measured from the bottom of the liquid upward, the corresponding depth of a point below the free surface is \( D - z \). Hence, from Eqn. (1.34), the differential horizontal force \( dF \) on an infinitesimally small rectangular strip of area \( dA = Wdz \) is:

\[
dF = pWdz = \rho g(D - z)Wdz.
\]

![Fig. E1.5 Variation of gas pressure with elevation.](image)
Integration from the bottom \((z = 0)\) to the top \((z = D)\) of the dam gives the total horizontal force:

\[
F = \int_0^D \rho g W (D - z) \, dz = \frac{1}{2} \rho g W D^2.
\]  \hfill (1.36)

**Horizontal pressure force on an arbitrary plane vertical surface.**

The preceding analysis was for a regular shape. A more general case is illustrated in Fig. 1.16, which shows a *plane* vertical surface of *arbitrary* shape. Note that it is now slightly easier to work in terms of a *downward* coordinate \(h\).

\[
F = \int_A p \, dA = \int_A \rho g h \, dA = \rho g A \frac{\int_A h \, dA}{A}.
\]  \hfill (1.37)

But the depth \(h_c\) of the centroid of the surface is defined as:

\[
h_c \equiv \frac{\int_A h \, dA}{A}.
\]  \hfill (1.38)
Chapter 1—Introduction to Fluid Mechanics

Thus, from Eqns. (1.37) and (1.38), the total force is:

\[ F = \rho gh_c A = p_c A, \]  

(1.39)

in which \( p_c \) is the pressure at the centroid.

The advantage of this approach is that the location of the centroid is already known for several geometries. For example, for a rectangle of depth \( D \) and width \( W \):

\[ h_c = \frac{1}{2} D \quad \text{and} \quad F = \frac{1}{2} \rho g WD^2, \]  

(1.40)

in agreement with the earlier result of Eqn. (1.36). Similarly, for a vertical circle that is just submerged, the depth of the centroid equals its radius. And, for a vertical triangle with one edge coincident with the surface of the liquid, the depth of the centroid equals one-third of its altitude.

Horizontal pressure force on a curved surface. Fig. 1.17(a) shows the cross section of a submerged surface that is no longer plane. However, the shape is uniform normal to the plane of the diagram.

In general, as shown in Fig. 1.17(b), the local pressure force \( p \, dA \) on an element of surface area \( dA \) does not act horizontally; therefore, its horizontal component must be obtained by projection through an angle of \( (\pi/2 - \theta) \), by multiplying by \( \cos(\pi/2 - \theta) = \sin \theta \). The total horizontal force \( F \) is then:

\[ F = \int_A p \sin \theta \, dA = \int_{A^*} p \, dA^*, \]  

(1.41)

in which \( dA^* = dA \sin \theta \) is an element of the projection of \( A \) onto the hypothetical vertical plane \( A^* \). The integral of Eqn. (1.41) can be obtained readily, as illustrated in the following example.
Example 1.6—Hydrostatic Force on a Curved Surface

A submarine, whose hull has a circular cross section of diameter $D$, is just submerged in water of density $\rho$, as shown in Fig. E1.6. Derive an equation that gives the total horizontal force $F_x$ on the left half of the hull, for a distance $W$ normal to the plane of the diagram. If $D = 8$ m, the circular cross section continues essentially for the total length $W = 50$ m of the submarine, and the density of seawater is $\rho = 1,026$ kg/m$^3$, determine the total horizontal force on the left-hand half of the hull.

**Solution**

The force is obtained by evaluating the integral of Eqn. (1.41), which is identical to that for the rectangle in Fig. 1.15:

$$F_x = \int_{A^*} p \, dA = \int_{z=0}^{z=D} \rho g W (D - z) \, dz = \frac{1}{2} \rho g W D^2. \quad (E1.6.1)$$

Insertion of the numerical values gives:

$$F_x = \frac{1}{2} \times 1,026 \times 9.81 \times 50 \times 8.0^2 = 1.61 \times 10^7 \text{ N.} \quad (E1.6.2)$$

Thus, the total force is considerable—about $3.62 \times 10^6 \text{ lb}_f$. 

---

**Fig. E1.6** *Submarine just submerged in seawater.*
Archimedes, ca. 287–212 B.C. Archimedes was a Greek mathematician and inventor. He was born in Syracuse, Italy, where he spent much of his life, apart from a period of study in Alexandria. He was much more interested in mathematical research than any of the ingenious inventions that made him famous. One invention was a “burning mirror,” which focused the sun’s rays to cause intense heat. Another was the rotating Archimedean screw, for raising a continuous stream of water. Presented with a crown supposedly of pure gold, Archimedes tested the possibility that it might be “diluted” by silver by separately immersing the crown and an equal weight of pure gold into his bath, and observed the difference in the overflow. Legend has it that he was so excited by the result that he ran home without his clothes, shouting “εἰρήκα, εἰρήκα”, “I have found it, I have found it.” To dramatize the effect of a lever, he said, “Give me a place to stand, and I will move the earth.” He considered his most important intellectual contribution to be the determination of the ratio of the volume of a sphere to the volume of the cylinder that circumscribes it. [Now that calculus has been invented, the reader might like to derive this ratio!] Sadly, Archimedes was killed during the capture of Syracuse by the Romans.


Buoyancy forces. If an object is submerged in a fluid, it will experience a net upward or buoyant force exerted by the fluid. To find this force, first examine the buoyant force on a submerged circular cylinder of height $H$ and cross-sectional area $A$, shown in Fig. 1.18.

![Fig. 1.18 Pressure forces on a submerged cylinder.](image)

The forces on the curved vertical surface act horizontally and may therefore be ignored. Hence, the net upward force due to the difference between the opposing pressures on the bottom and top faces is:

$$F = (p + \rho g H - p)A = \rho HAg,$$

(1.42)
which is exactly the weight of the displaced liquid, thus verifying *Archimedes’ law*, (the buoyant force equals the weight of the fluid displaced) for the cylinder. The same result would clearly be obtained for a cylinder of any uniform cross section.

![Diagram of buoyancy force for an arbitrary shape.](image)

*Fig. 1.19* Buoyancy force for an arbitrary shape.

Fig. 1.19 shows a more general situation, with a body of arbitrary shape. However, Archimedes’ law still holds since the body can be decomposed into an infinitely large number of vertical rectangular parallelepipeds or “boxes” of infinitesimally small cross-sectional area \( dA \). The effect for one box is then summed or “integrated” over all the boxes, and again gives the net upward buoyant force as the weight of the liquid displaced.

**Example 1.7—Application of Archimedes’ Law**

Consider the situation in Fig. E1.7(a), in which a barrel rests on a raft that floats in a swimming pool. The barrel is then pushed off the raft, and may either float or sink, depending on its contents and hence its mass. The cross-hatching shows the volumes of water that are displaced. For each of the cases shown in Fig. E1.7 (b) and (c), determine whether the water level in the pool will rise, fall, or remain constant, relative to the initial level in (a).

![Diagram of raft and barrel in swimming pool.](image)

*Fig. E1.7* Raft and barrel in swimming pool: (a) initial positions, (b) light barrel rolls off and floats, (c) heavy barrel rolls off and sinks. The cross-hatching shows volumes below the surface of the water.
Solution

Initial state. Let the masses of the raft and barrel be $M_r$ and $M_b$, respectively. If the volume of displaced water is initially $V$ in (a), Archimedes’ law requires that the total weight of the raft and barrel equals the weight of the displaced water, whose density is $\rho$:

$$ (M_r + M_b)g = V\rho g. \quad (E1.7.1) $$

Barrel floats. If the barrel floats, as in (b), with submerged volumes of $V_r$ and $V_b$ for the raft and barrel, respectively, Archimedes’ law may be applied to the raft and barrel separately:

Raft : $M_r g = V_r \rho g$, \quad Barrel : $M_b g = V_b \rho g$. \quad (E1.7.2)

Addition of the two equations (E1.7.2) and comparison with Eqn. (E1.7.1) shows that:

$$ V_r + V_b = V. \quad (E1.7.3) $$

Therefore, since the volume of the water is constant, and the total displaced volume does not change, the level of the surface also remains unchanged.

Barrel sinks. Archimedes’ law may still be applied to the raft, but the weight of the water displaced by the barrel no longer suffices to support the weight of the barrel, so that:

Raft : $M_r g = V_r \rho g$, \quad Barrel : $M_b g > V_b \rho g$. \quad (E1.7.4)

Addition of the two relations in (E1.7.4) and comparison with Eqn. (E1.7.1) shows that:

$$ V_r + V_b < V. \quad (E1.7.5) $$

Therefore, since the volume of the water in the pool is constant, and the total displaced volume is reduced, the level of the surface falls. This result is perhaps contrary to intuition: since the whole volume of the barrel is submerged in (c), it might be thought that the water level will rise above that in (b). However, because the barrel must be heavy in order to sink, the load on the raft and hence $V_r$ are substantially reduced, so that the total displaced volume is also reduced.

This problem illustrates the need for a complete analysis rather than jumping to a possibly erroneous conclusion.

\[\square\]
1.7 Pressure Change Caused by Rotation

Finally, consider the shape of the free surface for the situation shown in Fig. 1.20(a), in which a cylindrical container, partly filled with liquid, is rotated with an angular velocity $\omega$—that is, at $N = \omega/2\pi$ revolutions per unit time. The analysis has applications in fuel tanks of spinning rockets, centrifugal filters, and liquid mirrors.

![Diagram of a rotating cylinder with pressure changes](image)

Fig. 1.20  Pressure changes for rotating cylinder: (a) elevation, (b) plan.

Point O denotes the origin, where $r = 0$ and $z = 0$. After a sufficiently long time, the rotation of the container will be transmitted by viscous action to the liquid, whose rotation is called a forced vortex. In fact, the liquid spins as if it were a solid body, rotating with a uniform angular velocity $\omega$, so that the velocity in the direction of rotation at a radial location $r$ is given by $v_\theta = r\omega$. It is therefore appropriate to treat the situation similar to the hydrostatic investigations already made.

Suppose that the liquid element P is essentially a rectangular box with cross-sectional area $dA$ and radial extent $dr$. (In reality, the element has slightly tapering sides, but a more elaborate treatment taking this into account will yield identical results to those derived here.) The pressure on the inner face is $p$, whereas that on the outer face is $p + (\partial p/\partial r)dr$. Also, for uniform rotation in a circular path of radius $r$, the acceleration toward the center O of the circle is $r\omega^2$. Newton’s second law of motion is then used for equating the net pressure force toward O to the mass of the element times its acceleration:

$$\left( p + \frac{\partial p}{\partial r} dr - p \right) dA = \rho (dA dr) r\omega^2. \quad (1.43)$$

**Net pressure force**  **Mass**
Note that the use of a *partial* derivative is essential, since the pressure now varies in both the horizontal (radial) and vertical directions. Simplification yields the variation of pressure in the radial direction:

\[
\frac{\partial p}{\partial r} = \rho r\omega^2,
\]  

(1.44)

so that pressure *increases* in the radially outward direction.

Observe that the gauge pressure at all points on the interface is zero; in particular, \(p_O = p_Q = 0\). Integrating from points \(O\) to \(P\) (at constant \(z\)):

\[
\int_{p=0}^{p_P} dp = \rho \omega^2 \int_0^r r \, dr,
\]

\[
p_P = \frac{1}{2} \rho \omega^2 r^2.
\]

(1.45)

However, the pressure at \(P\) can also be obtained by considering the usual hydrostatic increase in traversing the path \(QP\):

\[
p_P = \rho gz.
\]

(1.46)

Elimination of the intermediate pressure \(p_P\) between Eqns. (1.45) and (1.46) relates the elevation of the free surface to the radial location:

\[
z = \frac{\omega^2 r^2}{2g}.
\]

(1.47)

Thus, the free surface is *parabolic* in shape; observe also that the density is not a factor, having been canceled from the equations.

There is another type of vortex—the *free* vortex—that is also important, in cyclone dust collectors and tornadoes, for example, as discussed in Chapters 4 and 7. There, the velocity in the angular direction is given by \(v_\theta = c/r\), where \(c\) is a constant, so that \(v_\theta\) is inversely proportional to the radial position.

**Example 1.8—Overflow from a Spinning Container**

A cylindrical container of height \(H\) and radius \(a\) is initially half-filled with a liquid. The cylinder is then spun steadily around its vertical axis \(Z-Z\), as shown in Fig. E1.8. At what value of the angular velocity \(\omega\) will the liquid just start to spill over the top of the container? If \(H = 1\) ft and \(a = 0.25\) ft, how many rpm (revolutions per minute) would be needed?
1.7—Pressure Change Caused by Rotation

Solution

From Eqn. (1.47), the shape of the free surface is a parabola. Therefore, the air inside the rotating cylinder forms a paraboloid of revolution, whose volume is known from calculus to be exactly one-half of the volume of the “circumscribing cylinder,” namely, the container.\(^8\) Hence, the liquid at the center reaches the bottom of the cylinder \textit{just as} the liquid at the curved wall reaches the top of the cylinder. In Eqn. (1.47), therefore, set \(z = H\) and \(r = a\), giving the required angular velocity:

\[
\omega = \sqrt{\frac{2gH}{a^2}}.
\]

For the stated values:

\[
\omega = \sqrt{\frac{2 \times 32.2 \times 1}{0.25^2}} = 32.1 \frac{\text{rad}}{\text{s}}, \quad N = \frac{\omega}{2\pi} = \frac{32.1 \times 60}{2\pi} = 306.5 \text{ rpm}. \quad \square
\]

---

\(^8\) Proof can be accomplished as follows. First, note for the parabolic surface in Fig. E1.8(b), \(r = a\) when \(z = H\), so, from Eqn. (1.47), \(\omega^2/2g = H/a^2\). Thus, Eqn. (1.47) can be rewritten as:

\[
z = H \frac{r^2}{a^2}.
\]

The volume of the paraboloid of air within the cylinder is therefore:

\[
V = \int_{z=0}^{z=H} \pi r^2 \, dz = \int_{z=0}^{z=H} \frac{\pi a^2 z}{H} \, dz = \frac{1}{2} \pi a^2 H,
\]

which is exactly one-half of the volume of the cylinder, \(\pi a^2 H\). Since the container was initially just half filled, the liquid volume still accounts for the remaining half.
1. Units conversion—E. How many cubic feet are there in an acre-foot? How many gallons? How many cubic meters? How many tonnes of water?

2. Units conversion—E. The viscosity $\mu$ of an oil is 10 cP and its specific gravity $s$ is 0.8. Reexpress both of these (the latter as density $\rho$) in both the lb$_m$, ft, s system and in SI units.

3. Units conversion—E. Use conversion factors to express: (a) the gravitational acceleration of 32.174 ft/s$^2$ in SI units, and (b) a pressure of 14.7 lb$_f$/in$^2$ (one atmosphere) in both pascals and bars.

4. Meteorite density—E. The Barringer Crater in Arizona was formed 30,000 years ago by a spherical meteorite of diameter 60 m and mass $10^6$ t (tonnes), traveling at 15 km/s when it hit the ground.\(^9\) (Clearly, all figures are estimates.) What was the mean density of the meteorite? What was the predominant material in the meteorite? Why? If one tonne of the explosive TNT is equivalent to five billion joules, how many tonnes of TNT would have had the same impact as the meteorite?

5. Reynolds number—E. What is the mean velocity $u_m$ (ft/s) and the Reynolds number $Re = \rho u_m D/\mu$ for 35 gpm (gallons per minute) of water flowing in a 1.05-in. I.D. pipe if its density is $\rho = 62.3$ lb$_m$/ft$^3$ and its viscosity is $\mu = 1.2$ cP? What are the units of the Reynolds number?

6. Pressure in bubble—E. Consider a soap-film bubble of diameter $d$. If the external air pressure is $p_a$, and the surface tension of the soap film is $\sigma$, derive an expression for the pressure $p_b$ inside the bubble. Hint: Note that there are two air/liquid interfaces.

---

7. Reservoir waterflooding—E. Fig. P1.7(a) shows how water is pumped down one well, of depth \( H \), into an oil-bearing stratum, so that the displaced oil then flows up through another well. Fig. P1.7(b) shows an enlargement of an idealized pore, of diameter \( d \), at the water/oil interface.

If the water and oil are just starting to move, what water inlet pressure \( p_w \) is needed if the oil exit pressure is to be \( p_o \)? Assume that the oil completely wets the pore (not always the case), that the water/oil interfacial tension is \( \sigma \), and that the densities of the water and oil are \( \rho_w \) and \( \rho_o \), respectively.\(^{10}\)

8. Barometer reading—M. In your house (elevation 950 ft above sea level) you have a barometer that registers inches of mercury. On an average day in January, you telephone the weather station (elevation 700 ft) and are told that the exact pressure there is 0.966 bar. What is the correct reading for your barometer, and to how many psia does this correspond? The specific gravity of mercury is 13.57.

![Fig. P1.9 Cylinder immersed in water and liquid A.](image)

9. Two-layer buoyancy—E. As shown in Fig. P1.9, a layer of an unknown liquid A (immiscible with water) floats on top of a layer of water W in a beaker. A completely submerged cylinder of specific gravity 0.9 adjusts itself so that its axis is vertical and two-thirds of its height projects above the A/W interface and one-third remains below. What is the specific gravity of A? Solve the problem two ways—first using Archimedes’ law, and then using a momentum or force balance.

![Fig. P1.10 U-tube with immiscible liquids.](image)

10. Differential manometer—E. The U-tube shown in Fig. P1.10 has legs of unequal internal diameters \( d_1 \) and \( d_2 \), which are partly filled with immiscible liquids of densities \( \rho_1 \) and \( \rho_2 \), respectively, and are open to the atmosphere at the top.

\(^{10}\) D.L. Katz et al., *Handbook of Natural Gas Engineering*, McGraw-Hill, New York, 1959, p. 57, indicates a wide range of wettability by water, varying greatly with the particular rock formation.
If an *additional* small volume $v_2$ of the second liquid is added to the right-hand leg, derive an expression—in terms of $\rho_1$, $\rho_2$, $v_2$, $d_1$, and $d_2$—for $\delta$, the amount by which the level at B will fall. If $\rho_1$ is known, but $\rho_2$ is unknown, could the apparatus be used for determining the density of the second liquid?

*Hints:* The lengths $h_A$, $h_B$, and $h_C$ have been included just to get started; they must not appear in the final result. After adding the second liquid, consider $h_C$ to have increased by a length $\Delta$—a quantity that must also eventually be eliminated.

![Fig. P1.11 Bubble rising in a closed cylinder.](image)

11. *Ascending bubble*—E. As shown in Fig. P1.11, a hollow vertical cylinder with *rigid* walls and of height $H$ is closed at both ends, and is filled with an *incompressible* oil of density $\rho$. A gauge registers the pressure at the top of the cylinder. When a small bubble of volume $v_0$ initially adheres to point A at the bottom of the cylinder, the gauge registers a pressure $p_0$. The gas in the bubble is ideal, and has a molecular weight of $M_w$. The bubble is liberated by tapping on the cylinder and rises to point B at the top. The temperature $T$ is constant throughout. Derive an expression in terms of any or all of the specified variables for the new pressure-gauge reading $p_1$ at the top of the cylinder.

12. *Ship passing through locks*—M. A ship of mass $M$ travels uphill through a series of identical rectangular locks, each of equal superficial (bird’s-eye view) area $A$ and elevation change $h$. The steps involved in moving from one lock to the next (1 to 2, for example) are shown as A–B–C in Fig. P1.12. The lock at the top of the hill is supplied by a source of water. The initial depth in lock 1 is $H$, and the density of the water is $\rho$.

(a) Derive an expression for the increase in mass of water in lock 1 for the sequence shown in terms of some or all of the variables $M$, $H$, $h$, $A$, $\rho$, and $g$.

(b) If, after reaching the top of the hill, the ship descends through a similar series of locks to its original elevation, again derive an expression for the mass of water gained by a lock from the lock immediately above it.

(c) Does the mass of water to be supplied depend on the mass of the ship if: (i) it travels only uphill, (ii) it travels uphill, then downhill? Explain your answer.
13. **Furnace stack—E.** Air \( (\rho_a = 0.08 \text{ lb}_m/\text{ft}^3) \) flows through a furnace where it is burned with fuel to produce a hot gas \( (\rho_g = 0.05 \text{ lb}_m/\text{ft}^3) \) that flows up the stack, as in Fig. P1.13. The pressures in the gas and the immediately surrounding air at the top of the stack at point A are equal.

What is the difference \( \Delta h \) (in.) in levels of the water in the manometer connected between the base B of the stack and the outside air at point C? Which side rises? Except for the pressure drop across the furnace (which you need not
worry about), treat the problem as one in hydrostatics. That is, ignore any frictional effects and kinetic energy changes in the stack. Also, neglect compressibility effects.

14. **Hydrometer—E.** When a hydrometer floats in water, its cylindrical stem is submerged so that a certain point X on the stem is level with the free surface of the water, as shown in Fig. P1.14. When the hydrometer is placed in another liquid L of specific gravity \( s \), the stem rises so that point X is now a height \( z \) above the free surface of L.

Derive an equation giving \( s \) in terms of \( z \). If needed, the cross-sectional area of the stem is \( A \), and when in water a total volume \( V \) (stem plus bulb) is submerged.

15. **Three-liquid manometer—E.** In the hydrostatic case shown in Fig. P1.15, \( a = 6 \text{ ft} \) and \( c = 4 \text{ ft} \). The specific gravities of oil, mercury, and water are \( s_o = 0.8 \), \( s_m = 13.6 \), and \( s_w = 1.0 \). Pressure variations in the air are negligible. What is the difference \( b \) in inches between the mercury levels, and which leg of the manometer has the higher mercury level? *Note:* In this latter respect, the diagram may or may not be correct.
16. **Pressure on Mt. Erebus**—M. On page 223 of the biography *Shackleton* (by Roland Huntford, Atheneum, New York, 1986), the Antarctic explorer’s colleague, Edward Marshall, is reported as having “. . . fixed the altitude [of Mt. Erebus] by hypsometer. This was simply a small cylinder in which distilled water was boiled and the temperature measured. It was then the most accurate known method of measuring altitude. The summit of Erebus turned out to be 13,500 feet above sea level.”

Assuming a uniform (mean) air temperature of $-5 \, ^\circ F$ (the summer summit temperature is $-30 \, ^\circ F$), and a sea-level pressure of 13.9 psia, at what temperature did the water boil in the hypsometer? At temperatures $T = 160, 170, 180, 190, 200, \text{ and } 210 \, ^\circ F$, the respective vapor pressures of water are $p_v = 4.741, 5.992, 7.510, 9.339, 11.526, \text{ and } 14.123 \, \text{psia}$.

17. **Oil and gas well pressures**—M. A pressure gauge at the top of an oil well 18,000 ft deep registers 2,000 psig. The bottom 4,000-ft portion of the well is filled with oil ($s = 0.70$). The remainder of the well is filled with natural gas ($T = 60 \, ^\circ F$, compressibility factor $Z = 0.80$, and $s = 0.65$, meaning that the molecular weight is 0.65 times that of air).

Calculate the pressure (psig) at (a) the oil/gas interface, and (b) the bottom of the well.

18. **Thrust on a dam**—E. Concerning the thrust on a rectangular dam, check that Eqn. (1.36) is still obtained if, instead of employing an upward coordinate $z$, use is made of a downward coordinate $h$ (with $h = 0$ at the free surface).

19. **Pressure variations in air**—M. Refer to Example 1.5 concerning the pressure variations in a gas, and assume that you are dealing with air at 40 $^\circ F$. Suppose further that you are using just the linear part of the expansion (up to the term in $z$) to calculate the absolute pressure at an elevation $z$ above ground level. How large can $z$ be, in miles, with the knowledge that the error amounts to no more than 1% of the exact value?

20. **Grand Coulee dam**—E. The Grand Coulee dam, which first operated in 1941, is 550 ft high and 3,000 ft wide. What is the pressure at the base of the dam, and what is the total horizontal force $F$ lb_f exerted on it by the water upstream?

21. **Force on V-shaped dam**—M. A vertical dam has the shape of a V that is 3 m high and 2 m wide at the top, which is just level with the surface of the water upstream of the dam. Use two different methods to determine the total force (N) exerted by the water on the dam.

22. **Rotating mercury mirror**—M. Physicist Ermanno Borra, of Laval University in Quebec, has made a 40-in. diameter telescopic mirror from a pool of

---

11 A more recent value is thought to be 12,450 feet.
mercury that rotates at one revolution every six seconds. (Air bearings eliminate vibration, and a thin layer of oil prevents surface ripples.)

By what value \( \Delta z \) would the surface at the center be depressed relative to the perimeter, and what is the focal length (m) of the mirror? The mirror cost Borra $7,500. He estimated that a similar 30-meter mirror could be built for $7.5 million. If the focal length were unchanged, what would be the new value of \( \Delta z \) for the larger mirror? *Hint*: the equation for a parabola of focal length \( f \) is \( r^2 = 4fz \).

23. *Oil and water in rotating container—E*. A cylindrical container partly filled with immiscible layers of water and oil is placed on a rotating turntable. Develop the necessary equations and prove that the shapes of the oil/air and water/oil interfaces are identical.

24. *Energy to place satellite in orbit—M*. “NASA launched a $195 million astronomy satellite at the weekend to probe the enigmatic workings of neutron stars, black holes, and the hearts of galaxies at the edge of the universe . . . The long-awaited mission began at 8:48 a.m. last Saturday when the satellite’s Delta–2 rocket blasted off from the Cape Canaveral Air Station.”

This “X-ray Timing Explorer satellite” was reported as having a mass of 6,700 lb and being placed 78 minutes after lift-off into a 360-mile-high circular orbit (measured above the earth’s surface).

How much energy (J) went directly to the satellite to place it in orbit? What was the corresponding average power (kW)? The force of attraction between a mass \( m \) and the mass \( M_e \) of the earth is \( GmM_e/r^2 \), where \( r \) is the distance of the mass from the center of the earth and \( G \) is the universal gravitational constant. The value of \( G \) is not needed in order to solve the problem, as long as you remember that the radius of the earth is \( 6.37 \times 10^6 \) m, and that \( g = 9.81 \text{ m/s}^2 \) at its surface.

25. *Central-heating loop—M*. Fig. P1.25 shows a piping “loop” that circulates hot water through the system ABCD in order to heat two floors of a house by means of baseboard fins attached to the horizontal runs of pipe (BC and DA). The horizontal and vertical portions of the pipes have lengths \( L \) and \( H \), respectively.

The water, which has a mean density of \( \bar{\rho} \) and a volume coefficient of expansion \( \alpha \), circulates by the action of natural convection due to a small heater, whose inlet and outlet water temperatures are \( T_1 \) and \( T_2 \), respectively. The pressure drop due to friction per unit length of piping is \( cu^2/D \), where \( c \) is a known constant, \( u \) is the mean water velocity, and \( D \) is the internal diameter of the pipe. You may assume that the vertical legs AB and CD are insulated, and that equal amounts of heat are dissipated on each floor.

Derive an expression that gives the volumetric circulation rate of water, \( Q \), in terms of \( c, D, \bar{\rho}, \alpha, g, L, H, T_1, \) and \( T_2 \).

---


26. Pressure at the center of the earth—M. Prove that the pressure at the center of the earth is given by \( p_c = \frac{3Mg_s}{8\pi R^2} \), in which \( g_s \) is the gravitational acceleration at the surface, \( M \) is the mass of the earth, and \( R \) is its radius. Hints: Consider a small mass \( m \) inside the earth, at a radius \( r \) from the center. The force of attraction \( mg_r \) (where \( g_r \) is the local gravitational acceleration) between \( m \) and the mass \( M_r \) enclosed within the radius \( r \) is \( GmM_r/r^2 \), where \( G \) is the universal gravitational constant. Repeat for the mass at the surface, and hence show that \( g_r/g_s = r/R \). Then invoke hydrostatics.

If the radius of the earth is \( R = 6.37 \times 10^6 \) m, and its mean density is approximately 5,500 kg/m\(^3\), estimate \( p_c \) in Pa and psi.

27. Soap film on wire rings—M. As shown in Fig. P1.27, a soap film is stretched between two wire rings, each of diameter \( D \) and separated by a distance \( H \). Prove that the radius \( R \) of the film at its narrowest point is:

\[
R = \frac{1}{6} \left( 2D + \sqrt{D^2 - 3H^2} \right).
\]

![Fig. P1.25 Central-heating loop.](image)

![Fig. P1.27 Soap film on two rings.](image)
You may assume that a section of the soap film is a circular arc, and that \( D \geq \sqrt{3}H \). What might happen if \( D \) is less than \( \sqrt{3}H \)?

Clearly stating your assumptions, derive an expression for the radius, in terms of \( D \) and \( H \). Is your expression exact or approximate? Explain.

28. **Treadmill stress test—M.** What power \( P \) is needed to resist a force \( F \) at a steady velocity \( V \)? In a treadmill stress test (Fig. P1.28), you have to keep walking to keep up with a moving belt whose velocity \( V \) and angle of inclination \( \theta \) are steadily increased. Initially, the belt is moving at 1.7 mph and has a grade (defined as \( \tan \theta \)) of 10%. The test is concluded after 13.3 min, at which stage the belt is moving at 5.0 mph and has a grade of 18%. If your mass is 163 lbm:

   (a) how many HP are you exerting at the start of the test, (b) how many HP are you exerting at the end of the test, and (c) how many joules have you expended overall?

29. **Bubble rising in compressible liquid—D.** A liquid of volume \( V \) and isothermal compressibility \( \beta \) has its pressure increased by an amount \( \Delta p \). Explain why the corresponding increase \( \Delta V \) in volume is given approximately by:

\[
\Delta V = -\beta V \Delta p.
\]

Repeat Problem P1.11, now allowing the oil—whose density and volume are initially \( \rho_0 \) and \( V_0 \)—to have a finite compressibility \( \beta \). Prove that the ratio of the final bubble volume \( v_1 \) to its initial volume \( v_0 \) is:

\[
\frac{v_1}{v_0} = 1 + \frac{\rho_0 g H}{p_0}.
\]

If needed, assume that: (a) the bubble volume is much smaller than the oil volume, and (b) \( \beta p_0 V_0 \gg v_1 \). If \( \rho_0 = 800 \text{ kg/m}^3, \beta = 5.5 \times 10^{-10} \text{ m}^2/\text{N}, H = 1 \text{ m,} \)

\( p_0 = 10^5 \text{ N/m}^2 \) (initial absolute pressure at the top of the cylinder), \( v_0 = 10^{-8} \text{ m}^3, \)

and \( V_0 = 0.1 \text{ m}^3 \), evaluate \( v_1/v_0 \) and check that assumption (b) above is reasonable.
**Problems for Chapter 1**

**51.** Methane

**Fig. P1.30 Well containing oil and methane.**

**30. Pressures in oil and gas well—M.** Fig. P1.30 shows a well that is 12,000 ft deep. The bottom \( H = 2 \),000-ft portion is filled with an incompressible oil of specific gravity \( s = 0.75 \), above which there is an \( h = 10 \),000-ft layer of methane \((\text{CH}_4; C = 12, H = 1)\) at 100 °F, which behaves as an ideal isothermal gas whose density is *not* constant. The gas and oil are static. The density of water is 62.3 \( \text{lb}_m/\text{ft}^3 \).

(a) If the pressure gauge at the top of the well registers \( p_A = 1 \),000 psig, compute the absolute pressure \( p_B \) (psia) at the oil/methane interface. Work in terms of symbols before substituting numbers.

(b) Also compute \( (p_C - p_B) \), the additional pressure (psi) in going from the interface B to the bottom of the well C.

**31. Soap film between disks—E (C).** A circular disk of weight \( W \) and radius \( a \) is hung from a similar disk by a soap film with surface tension \( \sigma \), as shown in Fig. P1.31. The gauge pressure inside the film is \( P \).
First, derive an expression for the angle $\theta$ in terms of $a$, $P$, $W$, and $\sigma$. Then obtain an equation that relates the radius of the neck $r$ to $a$, $P$, $W$, and $\sigma$. Assume that: (a) the excess pressure inside a soap film with radii of curvature $r_1$ and $r_2$ is $2\sigma(1/r_1 + 1/r_2)$, and (b) the cross section of the film forms a circular arc.

32. **Newspaper statements about the erg—E.** In the *New York Times* for January 18, 1994, the following statement appeared: “An erg is the metric unit scientists use to measure energy. One erg is the amount of energy it takes to move a mass of one gram one centimeter in one second.” (This statement related to the earthquake of the previous day, measuring 6.6 on the Richter scale, in the Northridge area of the San Fernando Valley, 20 miles north of downtown Los Angeles.)

Also in the same newspaper, there was a letter of rebuttal on January 30 that stated in part: “... This is not correct. The energy required to move a mass through a distance does not depend on how long it takes to accomplish the movement. Thus the definition should not include a unit of time.”

A later letter from another reader, on February 10, made appropriate comments about the original article and the first letter. What do you think was said in the second letter?

33. **Centroid of triangle—E.** A triangular plate held vertically in a liquid has one edge (of length $B$) coincident with the surface of the liquid; the altitude of the plate is $H$. Derive an expression for the depth of the centroid. What is the horizontal force exerted by the liquid, whose density is $\rho$, on one side of the plate?

34. **Blake-Kozeny equation—E.** The Blake-Kozeny equation for the pressure drop ($p_1 - p_2$) in laminar flow of a fluid of viscosity $\mu$ through a packed bed of length $L$, particle diameter $D_p$ and void fraction $\varepsilon$ is (Section 4.4):

$$\frac{p_1 - p_2}{L} = 150 \left( \frac{\mu u_0}{D_p^2} \right) \left[ \frac{(1 - \varepsilon)^2}{\varepsilon^3} \right].$$

(a) Giving your reasons, suggest appropriate units for $\varepsilon$.

(b) If $p_1 - p_2 = 75$ lb/in$^2$, $D_p = 0.1$ in., $L = 6.0$ ft, $\mu = 0.22$ P, and $u_0 = 0.1$ ft/s, compute the value of $\varepsilon$.

35. **Shear stresses for air and water—E.** Consider the situation in Fig. 1.8, with $h = 0.1$ cm and $V = 1.0$ cm/s. The pressure is atmospheric throughout.

(a) If the fluid is air at 20$^\circ$C, evaluate the shear stress $\tau_a$ (dynes/cm$^2$). Does $\tau$ vary across the gap? Explain.

(b) Evaluate $\tau_w$ if the fluid is water at 20$^\circ$C. What is the ratio $\tau_w/\tau_a$?

(c) If the temperature is raised to 80$^\circ$C, does $\tau_a$ increase or decrease? What about $\tau_w$?
36. True/false. Check true or false, as appropriate:\(^\text{14}\)

(a) When a fluid is subjected to a steady shear stress, it will reach a state of equilibrium in which no further motion occurs. T □ F □

(b) Pressure and shear stress are two examples of a force per unit area. T □ F □

(c) In fluid mechanics, the basic conservation laws are those of volume, energy, and momentum. T □ F □

(d) Absolute pressures and temperatures must be employed when using the ideal gas law. T □ F □

(e) The density of an ideal gas depends only on its absolute temperature and its molecular weight. T □ F □

(f) Closely, the density of water is 1,000 kg/m\(^3\), and the gravitational acceleration is 9.81 m/s\(^2\). T □ F □

(g) To convert pressure from gauge to absolute, add approximately 1.01 Pa. T □ F □

(h) To convert from psia to psig, add 14.7, approximately. T □ F □

(i) The absolute atmospheric pressure in the classroom is roughly one bar. T □ F □

(j) If \(\rho\) is density in g/cm\(^3\) and \(\mu\) is viscosity in g/cm s, then the kinematic viscosity \(\nu = \mu/\rho\) is in stokes. T □ F □

(k) For a given liquid, surface tension and surface energy per unit area have identical numerical values and identical units. T □ F □

(l) A force is equivalent to a rate of transfer of momentum. T □ F □

(m) Work is equivalent to a rate of dissipation of power per unit time. T □ F □

(n) It is possible to have gauge pressures that are as low as −20.0 psig. T □ F □

(o) The density of air in the classroom is roughly 0.08 kg/m\(^3\). T □ F □

(p) Pressure in a static fluid varies in the vertically upward direction \(z\) according to \(dp/dz = -\rho g_c\). T □ F □

---

\(^{14}\) Solutions to all the true/false assertions are given in Appendix B.
(q) At any point, the rate of change of pressure with elevation is \( \frac{dp}{dz} = -\rho g \), for both incompressible and compressible fluids.

(r) A vertical pipe full of water, 34 ft high and open at the top, will generate a pressure of about one atmosphere (gauge) at its base.

(s) The horizontal force on one side of a vertical circular disc of radius \( R \) immersed in a liquid of density \( \rho \), with its center a distance \( R \) below the free surface, is \( \pi R^3 \rho g \).

(t) For a vertical rectangle or dam of width \( W \) and depth \( D \), with its top edge submerged in a liquid of density \( \rho \), as in Fig. 1.15, the total horizontal thrust of the liquid can also be expressed as \( \int_0^D \rho ghW \, dh \), where \( h \) is the coordinate measured downwards from the free surface.

(u) The horizontal pressure force on a rectangular dam with its top edge in the free surface is \( F_x \). If the dam were made twice as deep, but still with the same width, the total force would be \( 2F_x \).

(v) A solid object completely immersed in oil will experience the same upward buoyant force as when it is immersed in water.

(w) Archimedes’ law will not be true if the object immersed is hollow (such as an empty box with a tight lid, for example).

(x) The rate of pressure change due to centrifugal action is given by \( \frac{\partial p}{\partial r} = \rho r^2 \omega \), in which \( \omega \) is the angular velocity of rotation.

(y) To convert radians per second into rpm, divide by \( 120\pi \).

(z) The shape of the free surface of a liquid in a rotating container is a hyperbola.

(A) The hydrostatic force exerted on one face of a square plate of side \( L \) that is held vertically in a liquid with one edge in the free surface is \( F \). If the plate is lowered vertically by a distance \( L \), the force on one face will be \( 3F \).
INDEX

A

Absolute pressure, 8
Aggregatively fluidized beds, 560, 566
fluid flow in, 569
particle flow in, 468
pressure distribution in, 567
Analogies, between momentum and heat transfer, 509
Angular momentum, 81
Angular velocity, 4, 39, 258, 355
Annular die, flow through, 322
Annular flow, 543, 552
A.P.I., degrees, 11
Archimedes, biographical sketch, 36
Archimedes’ law, 37
Archimedes number, 200
Axially symmetric irrotational flow, 378

B

Balances, energy, 9, 55, 61
mass, 9, 55, 57
momentum, 9, 55, 78
Basis or shape function, in finite-element methods, 681
Bearing, journal, 443
thrust, 443
flow in, using COMSOL, 448
Bernoulli, Daniel, biographical sketch, 68
Bernoulli’s equation, 67, 355, 384, 533
compressible flow, 161
generalized, 64
Bézier curves in COMSOL, 720
Bingham plastic fluids, 594, 600
in pipe flow 600
Blake-Kozeny equation, 207
Blasius equation, 129, 428, 493, 495, 511
Blasius solution for boundary layer flow, 425
Blow molding, 313
Blunt-nosed object, flow past, 358, 383
Bob-and-cup viscometer, 627
Body force, 56, 79
Body-force potential, 321
Boltzmann distribution, 649
Boltzmann’s constant, 131
Boolean operations in COMSOL, 722
Boundary, 9, 55
Boundary conditions, 293
Boundary layers, 414
application to turbulent jets, 513
dimensional analysis of, 430
laminar, 415
simplification of equations of motion for, 422
solution using COMSOL, 435
turbulent, 428
Boundary settings in COMSOL, 712, 724, 727
Bourdon-tube pressure gauge, 89
Brinkman equation with COMSOL, 729
Brownian motion, 129
Bubble caps, dynamics of, in distillation columns, 216
Bubble flow in vertical pipes, 543, 545
Bubbles, in fluidized beds, 560
formation at an orifice, 563
rise velocity of, 562, 572
Bubbles, rise velocity of, 531, 545
Bubbles, terminal velocity, 532
Buckingham Pi theorem, 227
Buffer region, 490
Buoyancy, 36
Burke-Plummer equation, 207

C

Cake, in a filter, 210
Calendering, 313, 401, 450
pressure distribution in, 455
Capillary pressure, in porous medium, 393
Capillary tube, for surface tension, 19
Capillary viscometer, 623
Caprock, 395
Carreau model, 599, 607
Cascade process in turbulence, 474
Centrifugal filter, 214
Centrifugal pump, 164, 189
Characteristic time, 596
Charge number, 644
“Choking” of the throat, 163
Churchill, S.W., Reynolds stress correlation, 496
interpolation between two asymptotic limits, 498
Coating a moving substrate, 461
Coating or spreading, 313
Coaxial cylinder rheometer, 627
Coefficient of contraction, 71
Coefficient of discharge, 73
Coefficient of thermal expansion, 12
Coions, 649
Colebrook and White equation, 136, 494
Commercial pipe, sizes, 138
Complex piping systems, 163
Composite object, COMSOL, 712, 723
Compressibility factor, 12
Compressibility, isothermal, 12
Compressible flow of gases, in a nozzle, 159
in a pipeline, 156
with COMSOL, 729
Compressive stress, 6
Computational fluid dynamics (CFD), 473, 671
applications in chemical engineering, 672
COMSOL, Inc., 703, 705
COMSOL Multiphysics, examples involving, boundary-layer flow, 435
die flow, non-Newtonian, 606
electroosmotic, 653, 657
jet flow and mixing, 505
lake flow, 373
lubricated bearing, 448
momentum diffusion, 307
multiphysics, 653, 657
orifice plate, 501
parallel-plate flow, 435
porous-medium flow, 705
screw extruder, 318
turbulent flow, 501, 505
COMSOL Multiphysics, capabilities of, 703
axes and grid settings, 710
Bézier curves, 720
Boolean operations, 722
boundary settings, 712, 724
composite object, 712, 723
documentation, 705
draw mode, 719
draw toolbar, 711
equations solvable by, 704
graphical user interface, 708
how to run, 705
interior boundaries
menus and toolbars, 709
mesh, 715, 716
model library
model navigator, 706
multiphysics, 653, 657
physics modes, 703
plot parameters, 718
postprocessing, 717
problems solvable by, 725
solving a problem, 717, 724
subdomain settings, 714, 724
surface plot, 716
Cone-and-plate viscometer, 328, 626
Connate water, 392
Conservation laws, 9, 55
Constitutive equations,
Bingham model, 594, 600
Carreau model, 599
generalized Newtonian fluids, 598
general viscoelastic fluids, 615
Maxwell model, 615
Newtonian fluids, 296, 595
power-law model, 599
White-Metzner model, 619
Contact force, 56, 79
Continuity equation, 59, 72, 267, 268
time-averaged, 477
Control surface, 55
for momentum transfer, 81
Convection of momentum, 81
Convective derivative, 266, 619
Converging/diverging nozzle, 159
Conversion factors, table of, inside front cover
Coriolis mass-flow meter, 95
Couette flow, 294, 312, 316, 328
in lubrication, 447
Counterions, 649
Critical pressure for compressible gas flow, 158, 163
Cross product, 251
Curl of a vector, 259
expressions for, 266
Curvature, 458, 735
Curved surface, change in pressure across, 18, 458
Cyclone separation, 219
Cylinder, flow past, computed by FlowLab, 195, 696
drag coefficient, 699
Cylindrical coordinates, 263
mass balance in, 268
momentum balances in, 322
solution of problems in, 322
D

d’Arcy’s law, 207, 388, 392, 706
Dam, force on, 32
Deborah number, 596
Debye–Huckel limit, 650
Debye length, 650
Deformation of a fluid element, 275, 357
Del (nabla) operator, in rectangular coordinates, 265
Density, 10
° A.P.I., 11
Derivative, definition of, 27, 257
Derivatives, 731
Derived quantities, 225
Diameter, hydraulic mean, 151
Die swell, 614
Dies, flow through, 313, 322 non-Newtonian, 606
Differential equations, solution of, 733
separation of variables, 267
spreadsheets, 466, 734
Differential mass balance, 267
Differential momentum balance, 271
Diffuser, in a nozzle, 159
Diffusion coefficient, 15
Diffusion in microchannels, 642
Diffusion of momentum, 307
Dilatant fluids, 593, 599, 603
Dimensional analysis, 224
Dimensional analysis of boundary layer flow, 430
Dimensionless groups, for drag force, 196
filtration, 224
flow through packed beds, 206
laminar sublayer, 230
pipe flow, 132, 134
pumps, 192
Dimensionless numbers, table of, 228
Dimensionless shear stress, 132, 491
Dimensions, 226
mass, length, and time, 10
Directional derivative, 252
Discharge coefficient, 73
Discretization, in numerical methods, 674
Dissipation, see frictional dissipation
Dissipation, turbulent, 499
transport equation for, 500
Distillation column, dynamics of bubble caps, 216
Dittus–Boelter equation, 511
Divergence of a vector, 254
expressions for, 265
Dot product, 250
Double-dot product, 597
Doublet, 384
Drag coefficient, 196
Drag coefficient on a flat plate, 415, 419, 428, 429, 432
Drag force, 194
Draw mode in COMSOL, 719
Draw toolbar in COMSOL, 711
Drawing or spinning, 312
Droplet, excess pressure inside, 18
Ducts, flow in noncircular, 150
Dyadic product, 619, 740
Dynamical similarity, 229

E

Eddies, 131, 474, 480
formation of, 475
Eddy diffusivity, 483
Eddy kinematic viscosity, 132, 482, 483, 484
correlation for, 486
determination of, 485
in turbulent jets, 518
Eddy thermal diffusivity, 483
Eddy transport, 481
Elastic modulus, 621
Elastic recoil, 617
Electrical double layer, 647, 648, 649
Electric charge, 644
flux of, 645
Electric field, 644
Electric potential, 646, 649, 650
Electrokinetic flow, 639, 664
Electrokinetic forces, 664
Electroosmosis, 647, 651
measurement of, 659
Electroosmosis in a microchannel (COMSOL), 653, 657
Electroosmotic flow around a particle, 653
Electroosmotic mobility, 647
Electrophoresis, 645, 664
Electrophoretic mobility, 645
Electrostatic precipitator, 202
Electroviscosity, 661
Energy balance, 55, 62, 598, 126, 128
Energy, conservation of, 9, 55
English units, 22
Entrance region between flat plates, 440
Eötvös number, in slug flow, 549
Equations of motion, 268, 281, 294, 322, 327
solutions of, 293
Equipment, visual encyclopedia of, 185
Equipotentials, 366
in microfluidics, 656, 658
Equivalent length of fittings, 154
Ergun equation, 206
Euler equation, 355, 397
Euler’s method, 733
Eulerian viewpoint, 267
Excel spreadsheets, 143, 145, 146, 150, 167, 454
Extrusion of polymer, 312
F

Falling-sphere viscometer, 202
Fanning friction factor, 132, 133, 135, 136, 137
Faraday’s constant, 649
FEMLAB—see under its new name, COMSOL Multiphysics
Fick’s law, 644
Film flow, 456
Film, in lubrication, 443
Filtrate, 210
Filtration, 210
  centrifugal, 213
  plate-and-frame, 210
  rotary-vacuum, 212
Finite-difference methods, 674
Finite-element methods, 680
Finite-volume methods, 676
Fittings, equivalent length, 154
Five-spot pattern, 391
Flooding, 555
Flow energy, 62
Flow, around sphere, 194
  in noncircular ducts, 150
  in open channels, 151
  past a flat plate, 415, 428
  through a porous medium, 207
  through packed beds, 204
FlowLab, examples involving,
  flow in pipe entrance, 687
  flow past a cylinder, 696
  sudden expansion, 690
  two-dimensional mixing, 692
FlowLab, CFD software, 682
  geometry panel, 684
  graphical user interface, 683
  mesh and solve panels, 685
  operation toolpad buttons, 683
  physics, boundary condition, and materials panels, 684
  reports and postprocessing panels, 686
Flow rates, 9
Flow rate, measurement of, 94
  by Coriolis meter, 95
  by orifice plate, 71
  by rotameter, 89
Flow regimes in two-phase flow,
  horizontal pipes, 541
  vertical pipes, 543
Fluent, Inc., 682
Fluid, definition of, 9
Fluid mechanics, laws of, 9
Fluidization, 215, 559
  aggregative, 560, 566
  particulate, 559
Fluidized bed, 559
  reaction in, 572
Flux, 8, 254
Force, 22
  as a rate of momentum transfer, 79
  on arbitrary surfaces, 33
  on dam, 32
  power for displacement of, 64
  units of, 21
Forced vortex, 39, 356
Form drag, 194
Fourier’s law, 256, 260
Fox, T.R.C., xvii, footnote
Free surface, 28, 33
  of rotating fluid, 39
Free vortex, 40, 220, 356
Friction factor, 124
  analogy with the Stanton number, 510
  as a dimensionless group, 132, 210
  in terms of Re, 135, 491
Friction-factor plot, 135
Frictional velocity, 489
Frictional dissipation, 63, 598
  noncircular ducts, 151
  open channels, 152
packed beds, 207
pipe flow, 126, 134
Froude number, in slug flow, 549
Fundamental dimensions, 225

G

Galerkin’s method, 681
Gas constant, values of, 12
Gas law, 11
Gas-lift pump, 550
Gas, pressure variations in, 31
Gas, underground storage of, 395
Gases, 5
  flow of compressible, 156, 159
  viscosity of, 131
Gauge pressure, 8
Gate valve, 154
gc, conversion factor, 22
General linear viscoelastic fluids, 615, 618
Generalized Maxwell model, 618
Geometrical shapes, 731
Geometrical similarity, 229
Globe valve, 154
Gradient of a scalar, 252
  expressions for, 265
Graphical interface, for
  COMSOL, 708

H

Hagen-Poiseuille law, 125
Harrison, D., bubble formation in fluidized beds, 565
Head, of fluid, 68
Head/discharge curve for centrifugal pump, 192
Heat transfer, analogy with momentum transfer, 509
Hookean solid, 616
Hoop stress in pipe wall, 139
Hydraulic mean diameter, 151
Hydraulically smooth pipe, 136
Hydrostatics, 26
multiple fluids, 30

Jet mixing, COMSOL
computation of, 505
FlowLab computation of, 692
Journal bearing, 443

Karamanev, D.G., method
for terminal velocities, 200
Kármán vortex street, 475, 697
$k/\varepsilon$ method for turbulent
flows, 499
with COMSOL, 726
with FlowLab, 690
Kinematic viscosity, 15, 512
Kinetic energy, 61, 67
for pipe flow, 127
Kinetic energy, turbulent, 499
transport equation for, 500
Kolmorogov limit, 474, 476
Kronecker delta, 251

Lagrangian viewpoint, 266
Lake flow, with COMSOL, 373
Lamb, Horace, feelings about
turbulence, 473
Laminar flow, friction in,
frictional dissipation, 126
friction factor for, 134
in a pipe, 122, 123
Laminar flow, unstable,
475, 700
Laminar sublayer, 155, 490
dimensional analysis of,
229
thickness of, 155, 493
Laminar velocity profile,
124, 155
Laplace, Pierre Simon, Mar-
quis de, biographical
sketch, 362
Laplace’s equation, 262
in irrotational flow, 362,
364, 379
for axially symmetric flow, 379
with COMSOL, 727
Laplacean operator, 262
expressions for, 266
Laws of fluid mechanics, 9
Leibnitz’s rule, 618 footnote, 736
Leung, L.S., bubble forma-
tion in fluidized beds, 565
Linear viscoelasticity, 615
Line source, 370
Liquids, 5
Lockhart/Martinelli
correlation, 539, 552
Logarithmic velocity profile,
487, 490
Lorentz force, 646
Loss angle, 621
Loss modulus, 621
Lubrication approximation,
444
Lubrication flow, with COM-
SOL, 448

Macintosh computer for
COMSOL, 706
Magnetic settling, 662
Manometer, 93
Mass, 21
conservation of, 9, 55
Mass balance, 55, 57
steady state, 57
Mass flow rate, 9
Mass velocity, 157
Material types, 591
MATLAB, xvi, 613, 703,
704, 706
Maxwell, James Clerk, bio-
graphical sketch, 616
Maxwell model, 615
Memory function, 618
Mesh refinement in COMSOL, 715, 725
Microfluidics, 639 chips for, 640
Microscale fluid mechanics, 640
Mist flow, 475
Mixing length, correlation for, 485
determination of, 484
Mixing-length theory, 481 for turbulent jets, 515
Model navigator, in COMSOL, 706
Moment of inertia, 90
Momentum, 78
angular, 90
balance, 55, 78
conservation of, 55, 79
diffusion of, 307
Momentum balance, for bubble formation at an orifice, 564
in film flow, 405
shell, 301
time-averaged, 478
Momentum transfer, by convection, 9, 81
by force, 79
in laminar flow, 129
in turbulent flow, 131
Momentum transfer, analogy with heat transfer, 509
Moody friction factor, 132

N
Natural gas, underground storage of, 395
Navier, Claude-Louis-Marie-Henri, biographical sketch, 281
Navier-Stokes equations, 278, 281
in microfluidics, 646
with COMSOL, 725
Needle valve, 154
Newton, Sir Isaac, biographical sketch, 131
law of viscosity, 124, 130
second law of motion, 21, 27
Newtonian fluid, 4, 14, 124, 130, 275, 276, 279, 591, 598
Nicklin, D.J., correlation for two-phase slug flow, 548
Nikuradse, pipe friction experiments, 136
Nonlinear simultaneous equations, 149, 166
Non-Newtonian flow in a die, with COMSOL, 606
viscosity profiles, 611
Non-Newtonian flows using COMSOL, 728
Non-Newtonian fluid, 4, 592
Normal stresses, 271, 276, viscoelastic, 613
Normal-stress difference, 613
No-slip boundary condition, 273
Nozzle, gas flow in, 159
Numerical methods for solving fluid mechanics problems, 673

O
Oldroyd derivative, 620
One-seventh power law, 493, 495
Open-channel flow, 151
Order-of-magnitude analysis, for boundary-layer flow, 423
for turbulent jets, 513
Ordinary differential equations, solution of, 733
Orifice, flow through, compressible, 159
incompressible, 70
Orifice-plate “meter,” 71
Orifice plate, COMSOL solution, 88, 501
pressure recovery, 504
Oscillatory shear, with COMSOL, 309
Ostwald-de-Waele model, 599
Packed beds, 204
Packed-bed reactor, pressure drop in, 208
Packed column, flooding of, 556
Paint films, leveling of, 463
Parabolic velocity profile, 124, 155
Parallel-plate rheometer, 627
Particle motion in microfluidic channels, 661
Particles, settling of, 199, 201, 222
Particulate phase ("emulsion"), in fluidization, 560
PC for COMSOL, 706
Péclet number, 643
Permeability, 208, 387, 391, 396
Permittivity, 646, 654
Physical properties, 10
Piezoelectric and piezoresistive effects in pressure transducer, 93
Piezometer, 93
Piezometric tube, 69, 93
Pipe fittings, pressure drop, 154
Pipe flow, Bingham plastic, 604
power-law fluid, 600
Pipe flow, pressure drop in, 123, 133, 139
Pipe roughness, 136
Pipeline, for gas, 156
Pipes, flow through, 120
Piping systems, 149, 163
Pitot tube, 74
Pitot-static tube, 74
Plate-and-frame filter, 210
Plot parameters in COMSOL, 718
Point source, 381
Poisson flow, 294, 312, 316
in lubrication, 447
Poisson’s equation, in lubrication, 445
Poisson’s equation, 
solution of, 
by COMSOL, 373, 727
by finite-element methods, 674
by finite-difference 
methods, 674
microfluidics, 646
Polymath, 150, 164
Polymer processing, 312, 450
Pores, flow through, 205
Porosity, 391, 396
Porous medium, flow 
through, 207, 566
single-phase, 364, 388, 390
two-phase, 390
with COMSOL, 728
Potential, for porous-
medium flow, 392
Potential energy, 61
Potential flow, 261, 361
Power,
for flowing stream, 64
for force displacement, 64
for pump, 64
for rotating shaft, 64
Power-law fluids, 599, 600
Power-law velocity profile, 495, 603
Prandtl hypothesis, 486
Prandtl, Ludwig, bio-
graphical sketch, 434
Prandtl-Taylor mixing length, 483
Prandtl-Taylor analogy, 510
Pressure, 6
absolute, 8
gauge, 8
Pressure as a function of 
height, 26
Pressure change caused by 
rotation, 39
Pressure distribution, in 
calendering, 454
in fluidized beds, 567
Pressure drop, across pipe 
 fittings, 154
in pipe flow, 123, 133, 139
Pressure drop in two-phase 
flow
horizontal pipes, 536
vertical pipes, 549, 552
Pressure forces on sub-
merged objects, 36
Pressure head, 68
Pressure measurement, 92
Pressure transducer, 93
Primary recovery of oil, 390
Projected area, 196
Pseudoplastic fluids, 593,
599, 603
Pump impeller, 91, 190
Pumps, centrifugal, 164, 189
positive displacement,
188, 189
Pumps in series and 
parallel, 193

R
Rabinowitsch equation, 624
Radius of curvature, 735
Rate laws, 57
Rate-of-deformation tensor,
279
Rate-of-strain tensor,
279, 596
invariants of, 597
Reaction in fluidized bed,
572
Reciprocating pumps, 188
Recirculation in sudden 
expansion,
in jet mixing, with 
FlowLab, 694
using FlowLab, 691
Rectangular coordinates, 249
mass balance in, 268
momentum balances in, 
272, 281
problems in, 294
Rectangular duct, flow 
through, 150, 294
Reference quantities, 430
Relaxation modulus, 618
Relaxation time, 616
Residual oil, 392
Reynolds analogy, 509
Reynolds experiment, 121
Reynolds number, 73,
122, 228
for boundary-layer flow,
415, 428
for drag force, 196
in microfluidics, 641
in pipe flow, 134, 135,
137, 149
Reynolds, Osborne, bio-
graphical sketch, 121
Reynolds stresses, 479
correlation for, 496
Rheometers, 625
Rheoplectic fluids, 594
Richardson-Zaki correlation,
222, 560
Rod-climbing effect, 614
Rotameter, 89
Rotary pumps, 189
Rotary-vacuum filter, 212
Rotating fluid, 39
Rotational flow, 356
Rotational rheometers, 625
Roughness, pipe, 136
Rough pipe, flow in, 136, 494

S
Saturation, in porous 
medium, 391
Scalars, 249
Schedule number for pipe,
137
Screw extruder, 313
with COMSOL, 318
Secondary recovery of oil, 
390
Sedimentation, 222
Separation of variables, 733
Settling of particles, 199,
201, 222
Shacham equation, for 
turbulent friction 
factor, 137, 150
Shear stress, 3, 6, 14,
271, 274
dimensionless groups for 
pipe flow, 225
distribution, 124, 300, 605
in pipe flow, 80, 123
models for, 129
Shear-thickening fluids, 593, 604
Shear-thinning fluids, 593, 599, 604
Shell momentum balance, 301
Shock, in gas flow, 159, 163
SI units, 21, 23
Sign convention for stresses, 271
Similar velocity profiles, 417
Simple shear, 592, 597, 615
Simpson’s rule, 733
Simultaneous nonlinear equations, 149, 166
Slug flow in vertical pipes, 543, 547
Slurry, 210
Smooth pipe, flow in, 490
Solenoidal flow, 256
Solids, 591, 616
Solution procedure, for viscous-flow problems, 293
Sound, speed of, 159, 163
Source in a uniform stream, 382
Specific gravity, 13
Sphere, drag force on, 194, 434
Sphere, flow past, 194, 386, 434
Spherical coordinates, 263
mass balance in, 268
momentum balances in, 283
solution of problems in, 327
Spherical-cap bubbles, 533
Sphericity, particle, 197, 198
Spinning of fibers, 325
Spray drier, 201
Spreadsheets, 143, 145, 146, 150, 167, 454, 734
Spreadsheet solution of differential equations, 734
Spring/dashpot model, 617
Stagnation flow, 369, 383
Stagnation point, 369, 383, 533
Static head, 68
Steady in the mean, 87, 421
Steady-state energy balance, 62
Steady-state mass balance, 59
Steady-state problems, 57
Stokes, Sir George Gabriel, biographical sketch, 198
Stokes’ law, 198
Storage modulus, 621
Strain rate, 592, 595, 598
for non-Newtonian flow in a die, 613
Stream function, 362, 364, 378
for boundary layers, 426
for turbulent jets, 516, 518
physical interpretation of, 364, 378
Streaming potential, 659
Streamlines, 57, 355, 366
in microfluidics, 656, 658
Stream tube, 62
Strength, of a doublet, 384
of a line source, 370
of a point source, 381
Stress and strain, for viscoelastic fluid, 622
Stress, compressive, 6
tensile, 5
Stress, sign convention for, 271
Stress relaxation, 617
Stress tensor, 274, 279, 595
Strong conservation form, 673, 676
Strouhal number, 699
Subdomain settings in COMSOL, 714, 724
Substantive derivative, 266
Sudden expansion, after orifice plate, 86
in a pipe, 88
solved by FlowLab, 690
Superficial velocity, 205
Supersonic velocity, 159, 162
Surface energy, 17
Surface plot in COMSOL, 716
Surface roughnesses, 136
Surface tension, 16
in thin-film flow, 456
methods for measuring, 19
Surface waves, 396
Surroundings, 9, 55
System, 9, 55

T

Tangential stresses, 6,
272, 274
Tank draining, 70
evacuation, 58
filling, 76
Taylor dispersion, 642
Taylor, Geoffrey Ingram, biographical sketch, 534
Taylor’s expansion, 31, 733
Tensile stress, 5
Tensors, 274, 279, 595, 740
“divergence” of, 741
“Laplacian” of, 741
Terminal velocities of spheres, 199
Karamanev method, 200
Tertiary recovery of oil, 390
Thermal diffusivity, 512
Thermal expansion, coefficient of, 12
Thin films, 456
Thixotropic fluids, 594
Thrust bearing, 443
Time-averaged continuity equation, 477
momentum balance, 478
Time-averaging, 476
Torque, 91, 191
power for rotation of, 64
Total head, 69
Transducer, for measuring pressure, 93
Transient problems, 57
Transient viscous diffusion of momentum
Transition flow, 121
Turbulence, 122, 124, 473
computation by the $k/\varepsilon$ method, 499
intensity of, 477
$k/\varepsilon$ method for, 499
mixing-length theory, 481
momentum transport in, 131, 480, 509
orifice-plate flow, 501
solved by COMSOL, 726
solved by FlowLab, 690, 692
velocity profiles, 155, 487, 490, 492
Turbulent boundary layers, 428
Turbulent core, 155, 229, 490, 492
Turbulent energy, 474, 499
dissipation rate $\varepsilon$, 499
fluctuations, 475, 476, 479
Turbulent jets, 513
axisymmetric, 519
plane, 514
Turbulent properties computed by COMSOL, dissipation, 503, 508
kinematic viscosity, 503, 507
kinetic energy, 503, 508
Turbulent transport, summary of, 483
Two-phase flow in porous media, 390
Two-phase flow in pipes, horizontal pipes, 536
vertical pipes, 543

Unstable laminar flow, 475, 700
Unsteady-state problems, 57
Usagi, R., interpolation between two asymptotic limits, 498

V
Valve, for pipeline, 154
Vanes, of centrifugal pump, 190
Variable-viscosity momentum balance, 284
Vector components, 250
Vector differentiation, 251
Vectors, 249
addition and subtraction, 250
dyadic product, 740
multiplication, 250
Velocity, 8
Velocity head, 68
Velocity, no-slip boundary condition, 293
Velocity of sound, 159, 162
Velocity potential, 260, 361, 364, 379
Velocity profiles, boundary-layer flow, 417, 418, 426, 428
calendering, 453, 463
concentric cylinders, 4
development in entrance region, 440
lubrication, 444
parallel plates, 14
pipe flow, 124, 155, 487, 490
turbulent flow, 487, 490, 492
turbulent jets, 517, 520
viscous flow, 124, 297, 305, 316, 324, 331
Vena contracta, 71
Viscoelastic fluids, 613, 618
constitutive equations, 613
phase relations, 622, 623
Viscometers, 625
Viscosity, 3, 13
eddy kinematic, 131, 483
kinematic, 15, 512
of gases, 131
Viscous dissipation function, 598
Viscous drag, 194
Viscous-flow problems, 292
Viscous modulus, 621
Visual encyclopedia of chemical engineering equipment, 185
Void fraction, 205
in two-phase flow, 536, 544, 549
Volumetric flow rate, 9
Volute chamber, 190
Von Kármán hypothesis, 487
Vortex, forced, 39, 356
free, 40, 356
Vortex formation during jet mixing, 506
Vortex lines, 355
Vortex shedding past a cylinder, 698
Vorticity, 260, 355, 358
for non-Newtonian flow in a die, 612
source term for, 373

W
Waterflooding, 391
Wave motion in deep water, 396
paths followed by particles, 399
Weight, 21
Weir, in distillation column, 217
Weissenberg effect, 614
Weissenberg, lectures of, 592
Weissenberg rheogoniometer, 328, 626
Wetted perimeter, 150
Weymouth equation, 157
White-Metzner model, 619
Work, 56, 61
Y

Yield stress, 594, 600

Z

Zajic, S.C., Reynolds stress
correlation, 496
Zero-shear boundary
condition, 294
Zero-shear viscosity, 599
Zeta potential, 647, 649, 651
Zoom extents, 375, 501, 506